FACH HOCHSCHULE LÜBECK

Hydrology
Evaporation

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Content

Objectives

To learn about ...

- types of evaporation
- estimation of evaporation
- measurement of evaporation

Basics to understand evaporation

- Physical background
- Hydrological Relevance
- Application

Evaporation
Relevance

Evaporation is a major component of the water balance accounting for about 60 to 70 % of the annual water balance in mid-latitudes and up to 99 % in arid climates. It is therefore of imminent importance to estimate evaporation for . . .

- irrigation projects
- management of water resources under changing climate
- management of reservoirs

Evaporation only plays a very minor in relation to floods. Only in very long rivers in arid zones evaporation has a significant effect on discharge along the river profile.

Physical background

Atmospheric parameters

with

$$\epsilon = \frac{\rho_v}{\rho_d}$$

$$q = \frac{\rho_v}{\rho}$$

$$rF = \frac{m}{m^*}$$

ϵ	Mixing Ratio
ρ_v	vapour density
ρ_d	density of dry air
ρ	$\rho = \rho_v + \rho_d$
q	specific moisture content
m^*	Mixing ratio
rF	specific moisture content

Barometric Pressure vs. Altitude

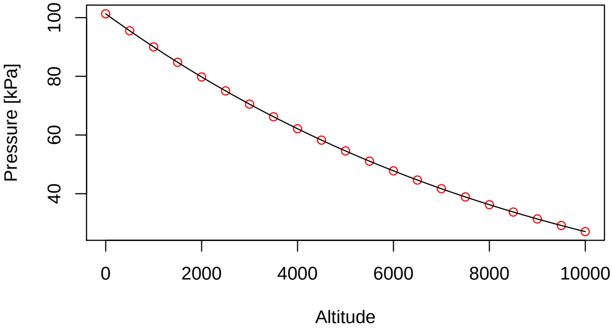
Equation:

$$P = 101.3 * \frac{(293 - 0.0065 * z)^{5.26}}{293} [kPa]$$

where

P	atmospheric pressure $[kPa]$
z	elevation above sea level $[m]$

Pressure profile Altitude vs. Pressure



International Barometric Function Altitude vs. Pressure

Equation:

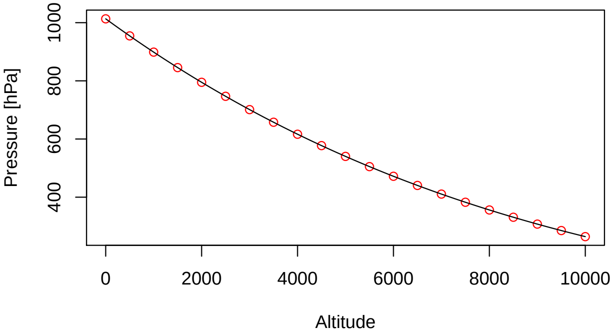
$$P = p_0 * \left[1 - \frac{0.0065\frac{K}{m} * h}{288.15 * K}\right]^{5.255} \quad [hPa]$$

$$p_0 = 1013.25\mathrm{hPa}$$

where

P	atmospheric pressure $[kPa]$
h	elevation above sea level $[m]$
K	temperature in $[K]$

Pressure with International Barometric Equation



Saturated Vapour Pressure Clasius-Clapeyron

$$e_s = 6.11 * 10^{(7.48*T/(237+T))} \, [hPa] \quad (1)$$

$$e_a = rH * e_s \, [hPa] \quad (2)$$

$$\quad (3)$$

e_s	saturated vapour pressure $[hPa]$
e_a	actual vapour pressure $[hPa]$
T	Temperature in degrees Celsius

Vapour pressure
Magnus-Formula

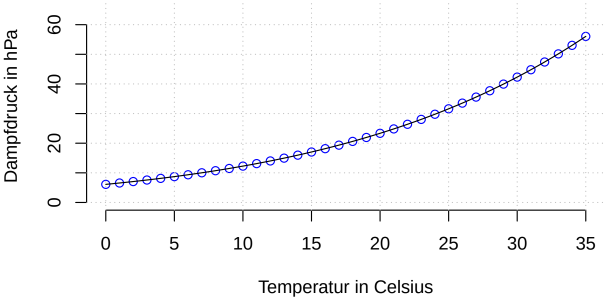


Figure: Vapour pressure calculated with Python

Programming in Python

```
import numpy as np
import matplotlib.pyplot as plt
# Create an array of 100
# linearly-spaced points from 0 to 35
T = np.linspace(0,35,350)
es = 6.11*10^(7.48*T/(237+T))
plt.plot(T,es)
plt.savefig('figure/Tes.png')
```

Saturated Vapour Pressure Clasius-Clapeyron

$$e_s = 0.6108 * exp(17.27 * T/(237.3 + T)) \, [kPa] \qquad (4)$$

e_s	saturated vapour pressure $[kPa]$
T	Temperature in degrees Celsius

Programming in R

```
T<-seq(0,35,1.0)
es<-0.6108*exp(17.27*T/(237+T))
plot(T,es,xlab="Temperature in Celsius",
ylab="Vapour pressure in kPa",
xlim=c(0, 35), ylim=c(0, 10),
pch=1, lty=1, col="blue", axes=FALSE)
axis(1, seq(0,35,5))
axis(2, seq(0,10,2))
grid (NULL,NULL, lty = 3, col = "grey")
curve(0.6108*exp(17.27*x/(237.3+x)), add = TRUE)
```

Vapour pressure
Dependency on temperature

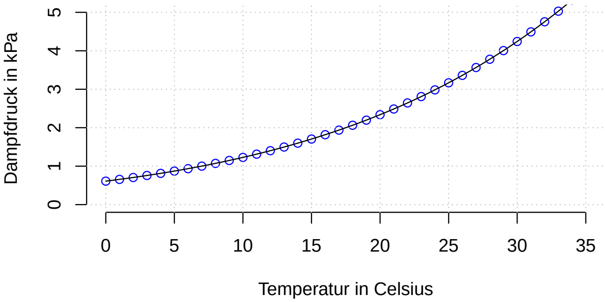


Figure: Vapour pressure as a function of temperature

Relative Humidity

Relative humidity rF is the percentage of actual humidity e_a in relation to potential humidity e_s at temperature T:

$$rF = e_a/e_s \ [\%]$$

The moisture **saturation deficit** d is defined as:

$$d = (e_s - e_a) \ \text{in} \ [hPa]$$

Psychrometric Constant

γ

Psychrometric constant γ is defined as:

$$\gamma = \frac{c_p * P}{\lambda * \epsilon} [kPa]$$
$$= 0.665 * 10^{-3} * P[kPa]$$

with

γ	Psychrometric constant []
c_p	specific heat at const. pressure $[MJ\,kg^{-1}\,{}^\circ C^{-1}]$
λ	latent heat of vaporization $[MJ\,kg^{-1}]$
ϵ	ratio of molecular weight of vapour dry air $[\epsilon = 0.622]$

Gradient of saturated vapour pressure curve Δ

$$\Delta = \frac{4098 * e_s}{237.3 + T} \tag{5}$$

Δ	gradient of sat. vapour pressure curve
e_s	saturated vapour pressure $[kPa]$
T	Temperature in degrees Celsius

Instruments and Methods

Measurement Methods
Water Balance

		Brief Description	Assumptions
Water budget measurements	Evaporation pan	Directly measures change in water level over time for a sample of open water in a “pan” with well-specified dimensions and siting.	Assumes relationship between measured evaporation from pans and actual evaporation from adjacent area can be calibrated, and calibration is transferable between locations and climates.
	Water balance of basin	The unmeasured difference between other measured components of the basin water balance, including incoming precipitation, surface and groundwater outflow, and soil water storage.	Assumes all other components of the basin water balance can be measured as spatial averages with sufficient accuracy for evaporation to be reliably calculated as the difference between them.
	Lysimetry	Measures change in weight of an isolated, preferably undisturbed, soil sample with overlying vegetation (if present) while measuring precipitation to and drainage from the sample.	Assumes the sample of soil and overlying vegetation on which measurements are made are representative in terms of soil water content and vegetation growth and vigor of the plot or field in question.
	Soil moisture depletion	Measures change in water content of a representative sample of undisturbed soil and vegetation while measuring precipitation and run-on/runoff and estimating deep drainage for the sample plot.	Assumes that soil water measuring devices (resistance blocks, tensiometers, neutron probes, time-domain reflectometers, capacitance sensors) adequately determine change in soil water, the effects of deep roots and sensor placement are small, and deep drainage can be estimated adequately.

Figure: Water Balance-based methods to determine evaporation, source: Shuttleworth (2008)

Measurement Methods

Vapour Transport

Water vapor transfer methods	**Bowen Ratio - Energy Budget**	Calculates evaporation as latent heat from the surface energy budget using the ratio of sensible to latent heat (Bowen ratio) derived from the ratio between atmospheric temperature and humidity gradients measured a few meters above vegetation.	Assumes the turbulent diffusion coefficient for sensible heat and latent heat are the same in the lower atmosphere in all conditions of atmospheric stability, and that plot-scale measurements of energy budget components (net radiation, soil heat) are representative of upwind conditions.
	Eddy correlation (also called eddy covariance)	Calculates evaporation as 20- to 60-minute time averages from the correlation coefficient between fluctuations in vertical windspeed and atmospheric humidity measured at high frequency (~10 Hz) at the same location, a few meters above vegetation.	Assumes only turbulent transfer of water vapor at sample point, and that corrections for water vapor transfer in turbulence at time scales less than ~0.1 seconds or greater than the selected averaging time are acceptable.
Components of evaporation	**Transpiration measurement by porometry or monitoring sap flow**	Porometry: measured from humidity increase in a chamber temporarily enclosing transpiring leaves/shoots. Sap Flow: measured from rate of sap flow in trunk, branches, or roots using heat as a tracer, with an estimate of the area of wood through which flow occurs.	Porometry assumes the enclosure of leaves and shoots in the chamber does not significantly alter transpiration rate. Sap Flow assumes installation of sensors does not alter sap flow rate, and cross-sectional area over which flow occurs can be determined accurately.
	Rainfall interception loss from tall vegetation	Measured as difference between cumulative rainfall above/below tall (usually forest) canopy. Requires careful below-canopy sampling with gauges/troughs that sample at spatial scale of canopy features, preferably randomly relocated after each measurement interval.	Assumes below-canopy sampling is adequate, a requirement rarely met for a typical 1-2 week measurement interval. It becomes feasible over several measurement intervals if gauges are regularly and randomly relocated.
	Soil evaporation	A small-scale, shallow implementation of lysimetry or soil moisture depletion methods for a near-surface soil sample below vegetation using several “microlysimeters” or sequential gravimetric multisampling.	Assumes the average of all small soil samples, regardless of their below-canopy location, are representative of the entire soil surface.

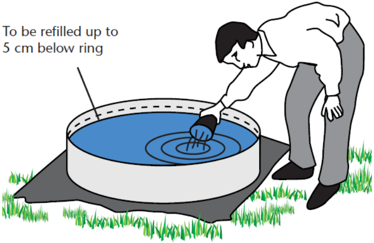
Figure: Vapour transport-based methods to determine evaporation, source: Shuttleworth (2008)

Measurement Methods
Large Scale

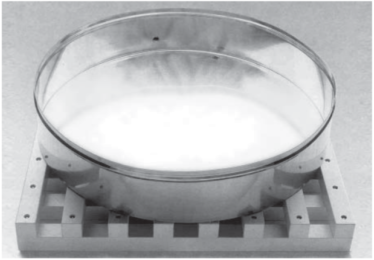
Large-scale evaporation	**Scintillometer measurements**	Uses theoretical relationship between sensible and latent heat fluxes and atmospheric scintillation introduced into a beam of electromagnetic radiation between source and detector by temperature and humidity fluctuations.	Applies strictly in an ideal turbulent field close, but not too close, to a surface with uniform aerodynamic roughness. However, field experiments suggest a worthwhile measurement is possible over a mixture of vegetation covers.
	Remote sensing estimates	Evaporation is deduced indirectly from the surface energy balance, with sensible heat calculated from the difference between air temperature and the temperature of the evaporating surface, along with an estimate of the aerodynamic exchange resistance between these two.	Assumes the "aerodynamic" surface temperature (that which controls sensible heat transfer from the surface), is the same as (or can be estimated from) the "radiometric" surface temperature (that which can be measured using an airborne or satellite radiometer).
	LIDAR (LIght Detection And Ranging) method	The local time-average vertical gradient of water vapor is sampled remotely using LIDAR. Local evaporation flux is calculated from this using similarity theory and supplementary measurements of friction velocity and atmospheric stability.	Assumes Monin-Obukov similarity theory applies and the supplementary measurements of friction velocity and atmospheric stability are locally applicable within the measurement field of the LIDAR.

Figure: Large scale methods to determine evaporation, source: Shuttleworth (2008)

Measurement Methods
Pans



United States Class A pan



GGI-3000 pan

Figure: Pans

Measurement of Evaporation
Class A pan

- Wood as support 15 cm
- Defined diameter 120.7 cm
- Water level 5-7 cm below rim
- Water depth ca. 25 cm
- Measurement of rainfall
- Wave protection
- Protection against animals (grid)

Measurement of Evaporation
Correction Factors

Class A pan	Case A: Pan placed in short green cropped area				Case B: Pan placed in dry fallow area			
RH mean (%) ®		low < 40	medium 40 - 70	high > 70		low < 40	medium 40 - 70	high > 70
Wind speed (m s^{-1})	Windward side distance of green crop (m)				Windward side distance of dry fallow (m)			
Light	1	.55	.65	.75	1	.7	.8	.85
< 2	10	.65	.75	.85	10	.6	.7	.8
	100	.7	.8	.85	100	.55	.65	.75
	1000	.75	.85	.85	1000	.5	.6	.7
Moderate	1	.5	.6	.65	1	.65	.75.	.8
2-5	10	.6	.7	.75	10	.55	.65	.7
	100	.65	.75	.8	100	.5	.6	.65
	1000	.7	.8	.8	1000	.45	.55	.6.
Strong	1	.45	.5	.6	1	.6	.65	.7
5-8	10	.55	.6	.65	10	.5	.55	.65
	100	.6	.65	.7	100	.45	.5	.6
	1000	.65	.7	.75	1000	.4	.45	.55
Very strong	1	.4	.45	.5	1	.5	.6	.65
> 8	10	.45	.55	.6	10	.45	.5	.55
	100	.5	.6	.65	100	.4	.45	.5
	1000	.55	.6	.65	1000	.35	.4	.45

- $E_{pot} = f_c * E_{pan}$
- correction factor depends on wind and humidity and surrounding

source: FAO, paper 24 65

Measurement of Evaporation
Example

Type of pan: Class A evaporation pan
Water depth in pan on day 1 = 150 mm
Water depth in pan on day 2 = 144 mm (after 24 hours)
Rainfall (during 24 hours) = 0 mm
K pan = 0.75
Formula: ET_o = K pan × E pan
Calculation: E pan = 150 - 144 = 6 mm/day
ET_o = 0.75 × 6 =4.5 mm/day

Measurement of Evaporation
Sunken pan

- $E_{pot} = f_c * E_{pan}$
- correction factor 0.83-0.98

source: John Rich, www.perrylakes.info

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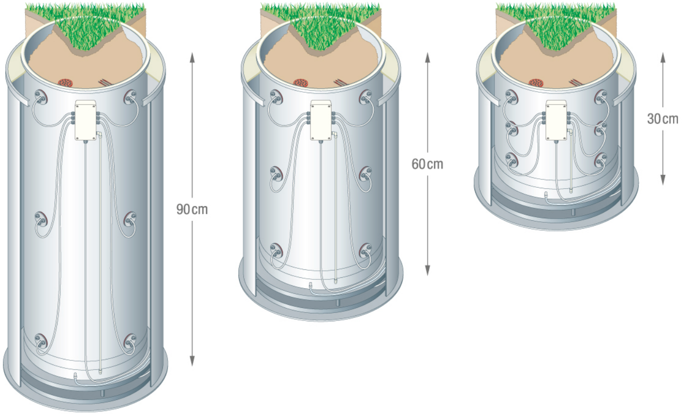
Measurement of Evaporation
Sunken pan

Location	Class *A* Pan	BPI sunken 6 ft, diameter, 2 ft. deep.	Colorado sunken 3 ft. square 2 ft. deep.	Screened sunken (young) 2 ft. diameter, 3 ft. deep.
Denver, Colorado (12 ft. pan 3 ft. deep)	0.67			
Fullerton, California (» »)	0.77	0.94	0.89	0.98
Ft. Mc Intosh, Texas (» »)	0.73			0.88
Falcon Dam, Texas (» »)	0.68			0.91
Dryden, Texas (» »)	0.73			0.96
Lake Elsinor, California	0.77			0.98
Red Bluff Reservoir, Texas	0.68			
Lake Okeechobee, Florida	0.81		0.98	
Lake Hefner, Oklahoma	0.69	0.91	0.83	0.91
Felt Lake, California	0.77	0.90	0.84	0.98
Lake Colorado City, Texas	0.72			

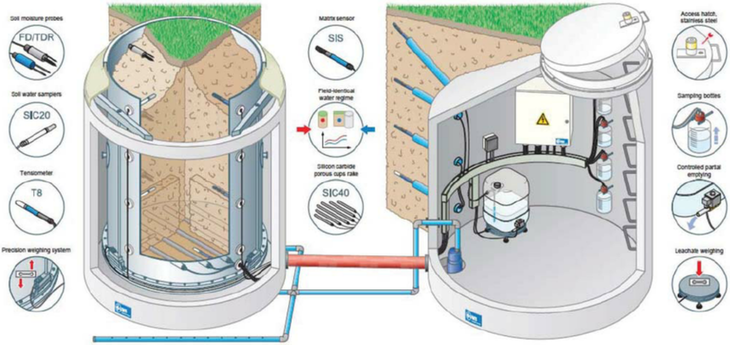
source: Gangopadhyaya

Measurement of Evaporation, Lysimeter



Lysimeter, UMS Meter

Measurement of Evaporation, Lysimeter Systems



Lysimeter, UMS Meter

Methods for the Estimation or Measurement of Evaporation

Factors

Evaporation depends on ...

- Air exchange, wind, advection
- Moisture saturation deficit of air
- Available energy as obtained from the energy balance

Key physical parameters are temperature and relative humidity. Wind speed is a parameter for air exchange and advection. Energy balances are only included in physical formulae, most empirical formulae rely on temperature, relative humidity and wind speed, some also on seasonal indices.

Estimation Methods - Overview

Calculation

- Dalton-type
- Empirical formulae
    - Blaney-Criddle
    - Turc
    - Thorntwaite
    - Hargreaves
- Water balance of basins
- Energy balance
- Bowen ratio
- Penman-Monteith
- Priestley-Taylor

Measurement

- Class A-pan, evaporimeters
- Profile method: T, v, e profiles
- Sap flux measurements
- Eddy flux correlation
- Lysimeters
- Lake studies
- Basin studies
- Isotope methods

Dalton Formula

One of the earliest approaches to estimate evaporation has been proposed by Dalton (1802) [1]. The formula proposed relates evaporation to saturation deficit $E_s - E_a$ and an advection term a.

$$E = f_D * (e_s - e_a)$$

with evaporation E in mm, saturated vapour pressure e_s and actual vapour pressure e_a. The factor f_D depends on wind speed.

Haude formula

Haude [2] proposed an adaption of the Dalton formula to regional conditions in Germany. The same author proposed advection term factors for Egypt. For the application of this formula measurements of temperature and relative humidity at 2 p.m. are needed.

$$E = f_H * [e_s(T_{14}) - rF_{14} * e_s(T_{14})] \, [mm/d] \quad (6)$$

For a green lawn in October $f_H = 0.2$.

Table: Haude Factors

Month	pasture	lawn	corn	deciduous trees	pine trees
Jan	0.2	0.2	0.11	0.01	0.08
Feb	0.2	0.2	0.11	0.01	0.04
Mar	0.25	0.23	0.11	0.04	0.14
Apr	0.29	0.24	0.17	0.10	0.35
May	0.29	0.29	0.21	0.23	0.30
Jun	0.28	0.29	0.24	0.28	0.34
Jul	0.26	0.28	0.25	0.32	0.31
Aug	0.25	0.26	0.26	0.26	0.25
Sep	0.23	0.23	0.21	0.17	0.20
Oct	0.22	0.20	0.18	0.10	0.13
Nov	0.20	0.20	0.11	0.01	0.07
Dec	0.20	0.20	0.11	0.005	0.05

DVWK formula

The German Association of Hydrologists recomends the DVWK method:

$$E = B * f(v) * (e_s - e_a) \, [mm/d] \tag{7}$$

The saturated vapour pressure e_s and actual vapour pressure e_a are given in hPa. B is a factor for surface roughness; $B = 0.135$. For rough surfaces with a fetch < 20 m, B can increase to $B = 0.25$. The wind function is simply given by $f(v)$ in m/s.

Boundary Layer Physics

Wind Profiles
Surface Roughness and Fetch

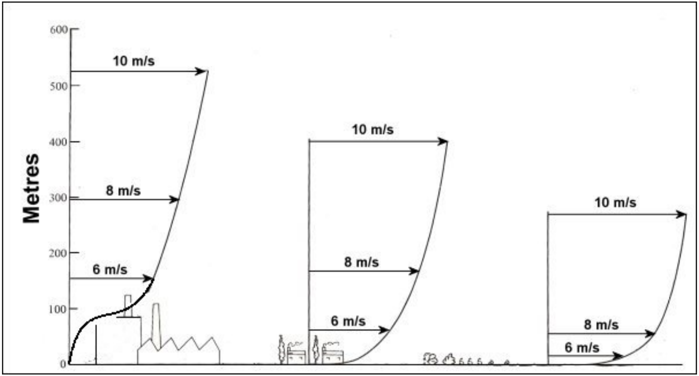


Figure: Wind profiles over different surfaces

Logarithmic Law

- Determining momentum transfer requires knowing velocity profile
- Flow of air over land or water – log velocity profile

$$u(z) = \frac{u^*}{k} \ln\left(\frac{z}{z_0}\right)$$

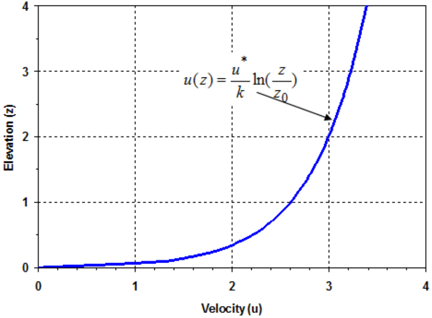
k Von Karman constant

z_0 Roughness height

$u^* = \sqrt{\tau_0 / \rho}$ Shear velocity

τ_0 Wall shear stress

$\frac{du}{dz} = \frac{u^*}{k}\frac{1}{z}$ Velocity gradient



Kinney & Merwade, 2007

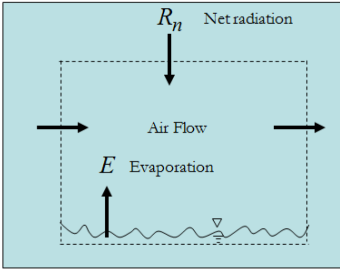
Derivation of Profile Method
Momentum Transfer

- Include transport of vapor away from water surface as function of:
  - Humidity gradient above surface
  - Wind speed across surface
- Upward vapor flux

$$\dot{m} = -\rho_a K_w \frac{dq_v}{dz} = \rho_a K_w \frac{q_{v_1} - q_{v_2}}{z_2 - z_1}$$

- Upward momentum flux

$$\tau = \rho_a K_m \frac{du}{dz} = \rho_a K_m \frac{u_2 - u_1}{z_2 - z_1}$$



$$\dot{m} = \tau \frac{K_w (q_{v_1} - q_{v_2})}{K_m (u_2 - u_1)}$$

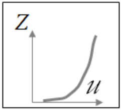
Kinney & Merwade, 2007

Derivation of Profile Method
Merge with Wind Profile

$$\dot{m} = \tau \frac{K_w (q_{v_1} - q_{v_2})}{K_m (u_2 - u_1)}$$

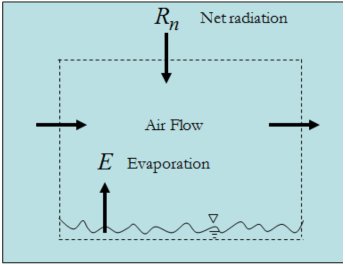
- Log-velocity profile

$$\frac{u}{u^*} = \frac{1}{k} \ln\left(\frac{Z}{Z_o}\right)$$



- Momentum flux

$$\tau = \rho_a \left[\frac{k(u_2 - u_1)}{\ln(Z_2/Z_1)}\right]^2$$



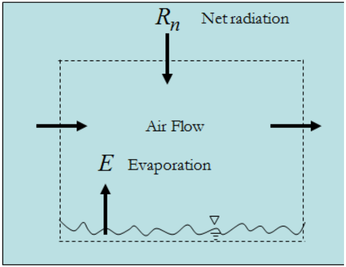
$$\dot{m} = \frac{K_w k^2 \rho_a (q_{v_1} - q_{v_2})(u_2 - u_1)}{K_m [\ln(Z_2/Z_1)]^2}$$

Thornthwaite-Holzman Equation

Kinney & Merwade 2007

Derivation of Profile Method
Solve and Simplify

$$\dot{m} = \frac{K_w k^2 \rho_a \left(q_{v_1} - q_{v_2}\right)\left(u_2 - u_1\right)}{K_m \left[\ln(Z_2/Z_1)\right]^2}$$



q_v and u

- Often only available at 1 elevation
- Simplifying

$$\dot{m} = \frac{0.622 k^2 \rho_a \left(e_{as} - e_a\right) u_2}{P\left[\ln(Z_2/Z_o)\right]^2}$$

$$\dot{m} = \rho_w A E \qquad e_a = \text{vapor pressure @ } Z_2$$

$$E_a = B\left(e_{as} - e_a\right)$$

$$B = \frac{0.622 k^2 \rho_a u_2}{P \rho_w \left[\ln(Z_2/Z_o)\right]^2}$$

prepared by Kinney & Merwade, 2007

Aerodynamic Method

$$E_a = B * (e_s - e_a) [mm/day] \quad (8)$$

$$e_s = 6.11 * 10^{(7.48*T/(237+T))} [hPa] \quad (9)$$

$$e_a = rH * e_s [hPa] \quad (10)$$

$$B = \frac{0.0102 * {u_2}^2}{ln(z_2/z_0)} \quad (11)$$

E_a	Evaporation by energy $[mm/d]$
e_s	saturated vapour pressure $[hPa]$
e_a	actual vapour pressure $[hPa]$
T	Temperature in degrees Celsius
u_2	wind speed at 2m height $[m/s]$
z_2	height 2m $[m]$
z_0	roughness height 0.01 - 0.06 m $[m]$

Temperature-based Methods

Empirical formulae
Blaney-Criddle

The Blaney-Criddle formula relies on day light percentage and temperature:

$$ET = p * (0.46 * T + 8.13)$$

with

ET	potentia evaporation in cm/month,
T	air temperature in degrees Celsius
p_i	annual daylight percentage of every month

Empirical formulae

Hamon

The Hamon formula relies on day length and temperature:

$$E_p = K * 0.165 * 216.7 * N * \left(\frac{e_s}{T + 273.3} \right) \quad (12)$$

$$e_s = 0.6108 * exp \left(\frac{17.27 * T}{T + 237.3} \right) \quad (13)$$

where

ET	potential evaporation in mm/day,
e_s	saturated vapour pressure mbar
k	empirical coefficient = 1.0 [-]
N	daylight hours [h]
e_s	saturated vapour pressure in [kPa]
T	average temperature in degree Celsius

Empirical formulae

Hargreaves

The Hargreaves formula relies on temperature and extraterrestrial radiation:

$$ET = 0.0023 * R_a * (T + 17.8) * TR^{0.50} \tag{14}$$

where

ET	potential evaporation in mm/day
R_a	extraterrestrial radiation $[MJm^2 day^1]$
T	average temperature in degree Celsius
TR	temperature range in degree Celsius

Energy Balance & Lake Evaporation

Energy Balance of a Lake

$$Q_0 = Q_s - Q_r + Q_a - Q_{ar} - Q_{bs} + Q_v - Q_e - Q_h - Q_w$$

where

Q_0	energy balance $[W/m^2]$
Q_s	energy from sun (short wave in) $[W/m^2]$
Q_r	energy reflected (short wave out) $[W/m^2]$
Q_a	energy from atmosphere (long wave in) $[W/m^2]$
Q_{ar}	energy reflected (long wave) $[W/m^2]$
Q_{bs}	energy emitted from water (long wave) $[W/m^2]$
Q_e	energy transferred by evaporation $[W/m^2]$
Q_h	energy transferred by sensible heat $[W/m^2]$
Q_v	energy advected $[W/m^2]$
Q_w	energy advected by evaporated water $[W/m^2]$

Radiation Balance

$$
\begin{aligned}
R_f &= Q_s - Q_{sr} + Q_a - Q_{ar} - Q_{bs} \\
&= Q_s - \alpha_s * Q_s + Q_a - \alpha_l * Q_a - Q_{bs} \\
&= Q_s * (1 - \alpha_s) + Q_a * (1 - \alpha_l) - Q_{bs}
\end{aligned}
$$

where

R_f	radiation balance $[W/m^2]$
Q_s	energy from sun (short wave in) $[W/m^2]$
Q_{sr}	energy reflected (short wave out) $[W/m^2]$
Q_a	energy from atmosphere (long wave in) $[W/m^2]$
Q_{ar}	energy reflected (long wave) $[W/m^2]$
Q_{bs}	energy emitted from water (long wave) $[W/m^2]$
α_s	short wave albedo [-]
α_l	long wave albedo [-]

Long Wave Emission

$$Q_{bs} = 0.97 * \sigma * T^4$$

Q_{bs}	energy emitted from water (long wave) $[W/m^2]$
σ	Stefan-Boltzmann Constant $5.67 * 10^{-8}$ $[W * m^{-2} * K^{-4}]$
T	Temperature in Kelvin [K]

The value 0.97 results from the emissivity of water (which is very high for long wave radiation, namely 0.97).

Advected Energy by Evaporation

$$Q_w = \frac{c_p * Q_e * (T_e - T_b)}{\lambda}$$

Q_w	energy advected by evaporation $[W/m^2]$
Q_e	latent energy transfer $[W/m^2]$
Q_h	sensible energy transfer $[W/m^2]$
c_p	specific heat of water 4186.8$[J/kgC]$
	latent energy of evaporation 2260$[kJ/kg]$
T_e	temperature of water
T_b	temperature of arbitrary datum

Bowen Ratio

$$B = \frac{Q_h}{Q_e}$$

B	Bowen ratio
Q_e	latent heat transfer
Q_h	sensible heat transfer

Energy for Evaporation

$$Q_e = \frac{Q_s - Q_r + Q_a - Q_{ar} - Q_{bs} - Q_0 + Q_v}{1 + B + c_p * (T_e - T_b)/\lambda}$$

Q_e	energy transferred by evaporation $[W/m^2]$
Q_s	energy from sun (short wave in) $[W/m^2]$
Q_r	energy reflected (short wave out) $[W/m^2]$
Q_a	energy from atmosphere (long wave in) $[W/m^2]$
Q_{ar}	energy reflected (long wave) $[W/m^2]$
Q_{bs}	energy emitted from water (long wave) $[W/m^2]$
Q_0	energy balance $[W/m^2]$
Q_v	energy advected $[W/m^2]$
B	Bowen ratio
c_p	specific heat of water 4186.8$[J/kgC]$
T_e, T_b	current and reference temperature of water
λ	latent energy of evaporation 2260$[kJ/kg]$

Energy for Evaporation

$$E = \frac{Q_e}{\rho * \lambda} = \frac{Q_s - Q_r + Q_a - Q_{ar} - Q_{bs} - Q_0 + Q_v}{\rho * \lambda * (1 + B) + c_p * (T_e - T_b)}$$

Q_s	energy from sun (short wave in) $[W/m^2]$
Q_r	energy reflected (short wave out) $[W/m^2]$
Q_a	energy from atmosphere (long wave in) $[W/m^2]$
Q_{ar}	energy reflected (long wave) $[W/m^2]$
Q_{bs}	energy emitted from water (long wave) $[W/m^2]$
Q_0	energy balance $[W/m^2]$
Q_v	energy advected $[W/m^2]$
B	Bowen ratio
c_p	specific heat of water 4186.8 $[J/kgC]$
T_e, T_b	temperature of water
ρ	density of water in kg/dm^3
λ	latent energy of evaporation 2260 $[kJ/kg]$

Energy, Radiation and Mass Transfer

Energy Method for Terrestrial Surface

$$E_r = R_n/(\lambda * \rho) \tag{15}$$
$$\lambda = 2.501 - 0.002361 * T[MJ/kg] = \tag{16}$$
$$R_n = R_i * (1 - \alpha) - R_e \tag{17}$$
$$R_e = e * \sigma * T_p^4 \tag{18}$$

E_r	Evaporation by energy $[mm/d]$
R_n	incoming radiation $[W/m^2]$
λ	latent heat of evaporation in $[W]$
T	Temperature in degrees Celsius
ρ	density of water in $[kg/dm^3]$
R_n, R_i	net and total incoming radiation $[W/m^2]$

Calculation of Incoming Radiation

$$R_G = \frac{24*60}{\pi} * G_{sc} * d_r * [\omega * sin(\phi) * sin(\delta) + cos(\phi) * cos(\delta) * sin(\omega_s)]$$

R_G	extraterrestrial radiation $[MJ\, m^{-2}\, day^{-1}]$
G_{sc}	solar constant $= 0.0820\, [MJ\, m^{-2}\, min^{-1}]$
d_r	inverse relative distance earth-sun
ω_s	sunset hour angle $[rad]$)
ϕ	latitude $[rad]$
δ	solar decimation $[rad]$

Converting decimal degrees to radians

$$rad = \frac{\pi}{180} * d$$

The variable d represents decimal degrees. For the northern hemisphere degrees in decimal degrees are positive for the southern hemisphere they are entered as negative values.

Table: Radians

degrees	rad
60	1.0472
50	0.8727
40	0.6981
30	0.5236
20	0.3491
10	0.1745
0	0.0000
-10	-0.1745
-20	-0.3491
-30	-0.5236
-40	-0.6981
-50	-0.8727
-60	-1.0472

Inverse Relative Distance Earth-Sun

$$d_r = 1 + 0.033 * cos\left(\frac{2\pi}{365} * J\right)$$

J is the Julian day commencing with J=1 on the first of January and ending with J=365 on the 31st of December.

Table: Inverse Distance Earth Sun

Month	Julian Day	relative inverse distance earth sun
January	1	1.033
February	31	1.028
March	60	1.017
April	91	1.000
May	121	0.9838
June	152	0.9714
July	182	0.967
August	213	0.9714
September	244	0.9838
October	274	1.000
November	305	1.017
Dezember	335	1.029
New Year	365	1.033

Inverse Relative Distance Earth-Sun

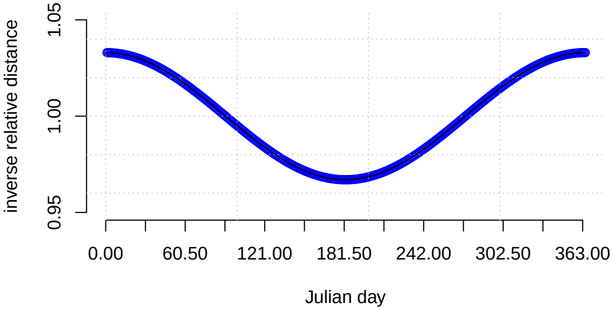


Figure: Relative inverse distance earth-sun

Solar Declination

$$\delta \quad = \quad 0.409 * sin\left(\frac{2*\pi}{365} * J - 1.39\right)$$

J is the Julian day commencing with J=1 on the first of January and ending with J=365 on the 31^{st} of December.

Table: Solar Declination

Month	Julian Day	solar declination
January	1	−0.401
February	31	−0.309
March	60	−0.143
April	91	0.07181
May	121	0.2613
June	152	0.385
July	182	0.403
August	213	0.3113
September	244	0.133
October	274	−0.07527
November	305	−0.2693
Dezember	335	−0.3862
New Year	365	−0.4023

Solar Declination

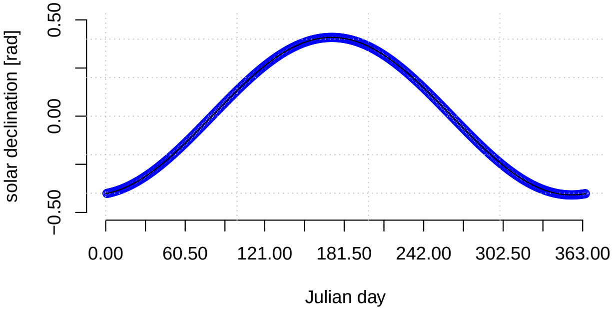


Figure: Solar declination

Sunset Hour Angle

$$\omega = arccos\left[-tan(\phi) * tan(\delta)\right] \quad (19)$$

$$\omega = \frac{\pi}{2} - arctan\left(\frac{-tan(\phi) * tan(\delta)}{\sqrt{X}}\right) \quad (20)$$

$$N = \frac{24}{\pi} * \omega_s \quad (21)$$

N	daylength
ω	sunset hour angle (radians)
ϕ	latitude (radians)
δ	declination (radians)
X	$1 - [tan(\phi)]^2 * [tan(\delta)]^2$
N	Day length (max. sunshine hours

Table: Sunshine hours

Month	Julian Day	sunshine hours
January	1	7.240
February	31	8.525
March	60	10.476
April	91	12.758
May	121	14.879
June	152	16.520
July	182	16.790
August	213	15.505
September	244	13.415
October	274	11.206
November	305	9.023
Dezember	335	7.463
New Year	365	7.220

Sunshine Hours

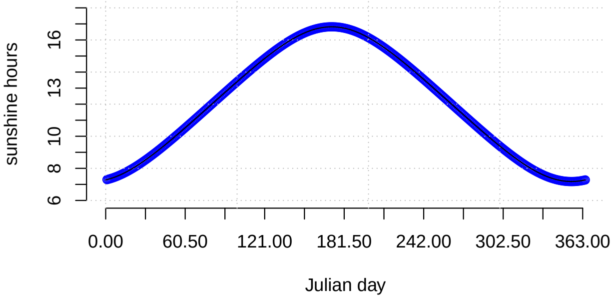


Figure: Sunshine hours

Extraterrestrial Radiation

$$R_a = \frac{24 * (60)}{\pi} * G_{sc} * d_r * [\omega_s * sin(\phi) + cos(\phi) * cos(\delta) * sin(\omega_s)]$$

where

R_a	extraterrestrial radiation $[MJm^{-2}day^{-1}]$
G_{sc}	solar constant $= 0.0820$ $[MJm^{-2}min^{-1}]$
d_r	inverse relative distance earth-sun
ω_s	sunset hour angle (radians)
ϕ	latitude (radians)
δ	declination (radians)

Extraterrestrial Radiation

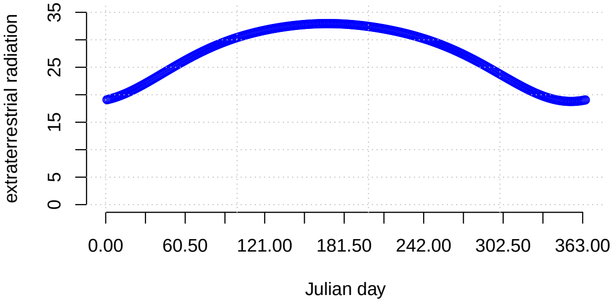


Figure: Extraterrestrial radiation

Conversion factors

Table: convert

	$\frac{MJ}{m^2*d}$	$\frac{J}{m^2*d}$	$\frac{cal}{m^2*d}$	$\frac{W}{m^2}$	$\frac{mm}{d}$
$\frac{MJ}{m^2*d}$	1	100	23.09	11.6	**0.408**
$\frac{cal}{m^2*d}$	$4.1868 * 10^{-2}$	4.1868	1	0.485	0.171
$\frac{W}{m^2}$	0.0864	8.64	2.06	1.0	0.035
$\frac{mm}{d}$	2.45	245	28.4	28.4	1.000

Multiplication with 0.408 converts $\frac{MJ}{m^2*d}$ to $\frac{mm}{d}$ which is the equivalent amount of water $\frac{mm}{d}$ that can be evaporated by the energy $\frac{MJ}{m^2*d}$.

Short-Wave Radiation on Earth
After travelling through atmosphere and clouds

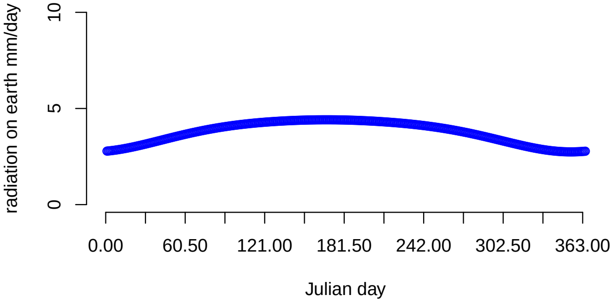


Figure: Short wave radiation at earth surface in mm/d

Net Long Wave Radiation

$$R_l = (0.34 - 0.14 * \sqrt{e_a}) * \sigma * \left[\frac{T^4_{max,K} - T^4_{min,K}}{2}\right] * (1.35 - (n/N) * 0.35)$$

$$e_a = 0.6108 * exp(17.27 * T/(237.3 + T))$$

R_{nl}	Net long wave outgoing radiation $[MJm^{-2}day^{-1}]$
e_a	actual vapour pressure $[kPa]$
$T_{min,K}$	Temperature in Kelvin (min. and max. measured during day)

Soil Heat Flux

$$G = c_s * \left[\frac{T_i - T_{i-1}}{\Delta t} \right] * \Delta z$$

G	soil heat flux in $[MJm^{-2}day^{-1}]$
c_s	soil heat capacity $[MJm^{-3\circ}C^{-1}]$
T_i	air Temperature in $^\circ$ *Celsius*
Δt	time step $[day]$
Δz	soil depth $[m]$

Combined Equations

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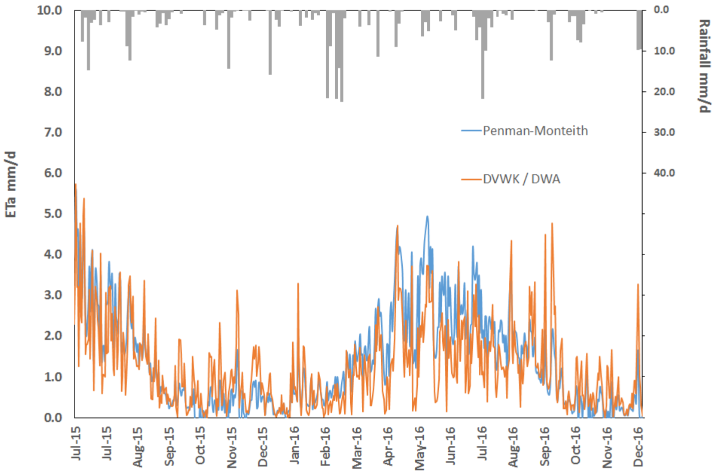
Seasonal Variation Evaporation



Combined Method
Energy and Aerodynamic Term

$$E_c = \left(\frac{\Delta}{\Delta + \gamma}\right) * E_r + \left(\frac{\gamma}{\Delta + \gamma}\right) * E_a \, [mm/d] \quad (22)$$

$$\Delta = \frac{4098 * e_s}{237.3 + T} \quad (23)$$

E_c	Evaporation from combined method [hPa]
E_a	Evaporation from aerodynamic method [hPa]
E_r	Evaporation from energy method
Δ	gradient of sat. vapour pressure curve
γ	psychrometric constant 66.8 $Pa/(^\circ C)$
T	Temperature in degrees Celsius

Combined Method
Energy Term

$$E_r = \left(0.25 + 0.5 * \frac{n}{N}\right) * S_0 - \left(0.9 * \frac{n}{N} + 0.1\right)$$
$$(0.34 - 0.14 * \sqrt{e_d}) * \sigma * T^4$$

E_r	Evaporation from energy method
n	sunshine hours
N	potential sunshine hours
e_d	vapour pressure
σ	Boltzmann-Constant
T	Temperature in degrees Celsius

Priestley & Taylor

$$
\begin{aligned}
E_{PT} &= \alpha * \left(\frac{\Delta}{\Delta + \gamma}\right) * R_n \\
R_n &= R_s * (1 - \alpha) - R_e \\
R_e &= e * \sigma * T_p^4 \\
E_r &= R_n / (\lambda * \rho) \\
\lambda &= 2.501 - 0.002361 * T [MJ/kg]
\end{aligned}
$$

E_{PT}	Evaporation by Priestley & Taylor $[mm/d]$
α	1.3
R_n	net radiation $[W/m^2]$
R_s	total short wave incoming radiation $[W/m^2]$
λ	latent heat of evaporation in $[W]$
T	Temperature in degrees Celsius
ρ	density of water in $[kg/dm^3]$

Makkink

$$ET = \alpha * \left(\frac{\Delta}{\Delta + \gamma} * \frac{R_s}{\lambda} \right) - \beta$$

ET	Evaporation by Makkink $[mm/d]$
R_s	solar radiation in $[MJm^{-2}day^{-1}]$
α	0.61
β	0.12
λ	latent heat of evaporation in $MJkg^{-1}$
Δ	slope of the vapour pressure curve in $[kPa\,^{\circ}C^{-1}]$
γ	psychrometric constant $[kPa\,^{\circ}C^{-1}]$

Doorenbos & Pruitt

$$ET = a + b * \left(\frac{\Delta}{\Delta + \gamma} * \frac{R_s}{\lambda} \right)$$

$$a = 1.066 - 0.13 * 10^{-2} * rH + 0.45 * u_z - 0.2 * 10^{-3} * rH * u_z$$
$$-0.315 * 10^{-4} * rH^2 - 0.11 * 10^{-2} * u_z^2$$

$$b = -0.3$$

ET	Evaporation $[mm/d]$
R_s	solar radiation in $[MJm^{-2}day^{-1}]$
λ	latent heat of evaporation in $MJkg^{-1}$
Δ	slope of the vapour pressure curve in $[kPa^\circ C^{-1}]$
γ	psychrometric constant $[kPa^\circ C^{-1}]$
u_z	wind speed at 2m in $[m/s]$
rH	relative humidity in $[\%]$

Abtew

$$ET = \alpha * \left(\frac{R_s}{\lambda}\right)$$

ET	Evaporation $[mm/d]$
α	0.53
R_s	solar radiation in $[MJm^{-2}day^{-1}]$
λ	latent heat of evaporation in $MJkg^{-1}$

Penman-Monteith Method, FAO version

$$E_{T_0} = \frac{0.408 * (R_n - G) + \gamma * \frac{900}{T+273} * u_2 * (e_s - e_a)}{\Delta + \gamma * (1 + 0.34 * u_2)} \quad (24)$$

E_{T_0}	reference evaporation $[mm/d]$
R_n	net radiation at the crop surface $[MJ/m^{-2} day{-1}]$
G	soil heat flux $[MJ/m^{-2} day{-1}]$
T	mean daily temperature at 2m height $[^\circ Celsius]$
u_2	wind speed at 2m height $[ms^{-1}]$
e_s	saturation vapour pressure $[kPa]$
e_a	actual vapour pressure $[kPa]$
Δ	slope of vapour pressure curve $[kPa^\circ Celsius^{-1}]$
γ	psychrometric constant $[kPa^\circ Celsius^{-1}]$

Actual Evaporation and Reduction Functions

Actual Evaporation
Reduction by Resistance

$$\lambda * ETo = \frac{\frac{\Delta}{\lambda} * (R_n - G) + \frac{\rho * c_p}{\lambda} * \frac{(e_s - e_a)}{r_a}}{\Delta + \gamma * \left(1 + \frac{r_s}{r_a}\right)} \quad (25)$$

where as in [3]

ETo	reference evaporation $[mm/d]$
R_n	net radiation at the crop surface $[MJ/m^{-2} day{-1}]$
G	soil heat flux $[MJ/m^{-2} day{-1}]$
T	mean daily temperature at 2m height $[^{\circ} Celsius]$
u_2	wind speed at 2m height $[ms^{-1}]$
e_s	saturation vapour pressure $[kPa]$
e_a	actual vapour pressure $[kPa]$
Δ	slope of vapour pressure curve $[kPa^{\circ} Celsius^{-1}]$
γ	psychrometric constant $[kPa^{\circ} Celsius^{-1}]$

Actual Evaporation
Reduction by Resistance

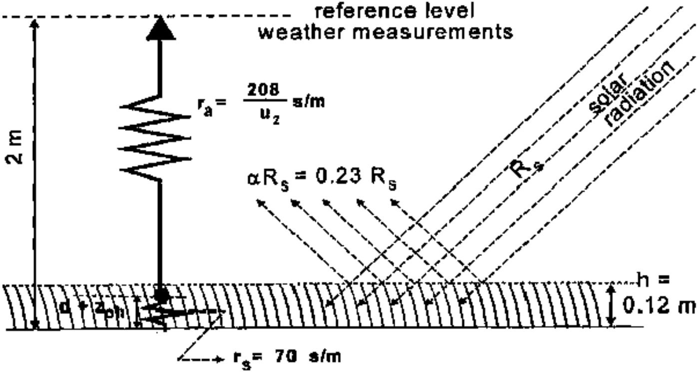


Figure: Reduction of evaporation by resistance

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Actual Evaporation
Reduction with Crop Coefficient

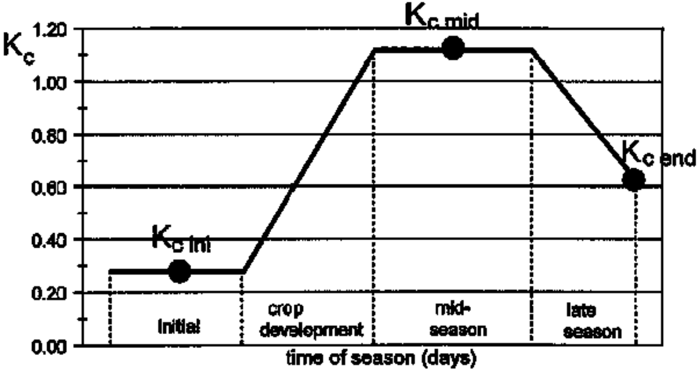

Figure: Reduction of evaporation by crop coefficient

Actual Evaporation
Reduction after Infiltration

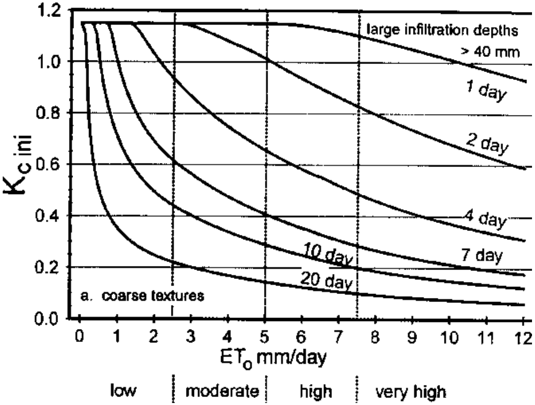

Figure: Reduction of evaporation by crop coefficient

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Actual Evaporation
Soil Moisture Reduction

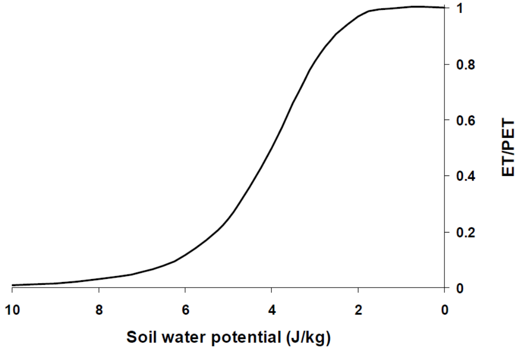


Figure: Reduction of evaporation by limited soil moisture

Actual Evaporation
Soil Function

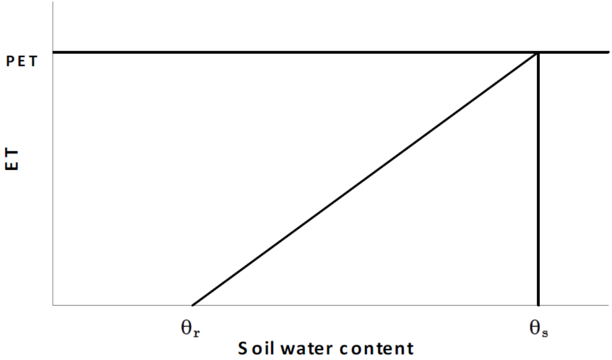


Figure: Reduction of evaporation by limited soil moisture

Actual Evaporation
Water Table Reduction

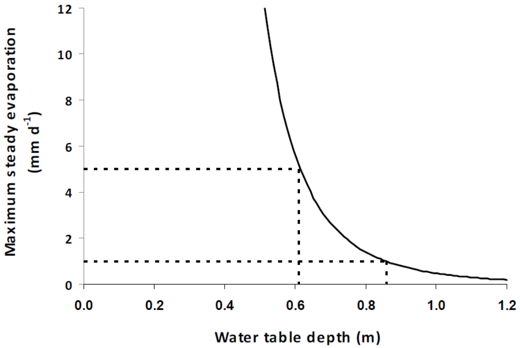


Figure: Reduction of evaporation by deep water table

Bagrov-Gugla Method

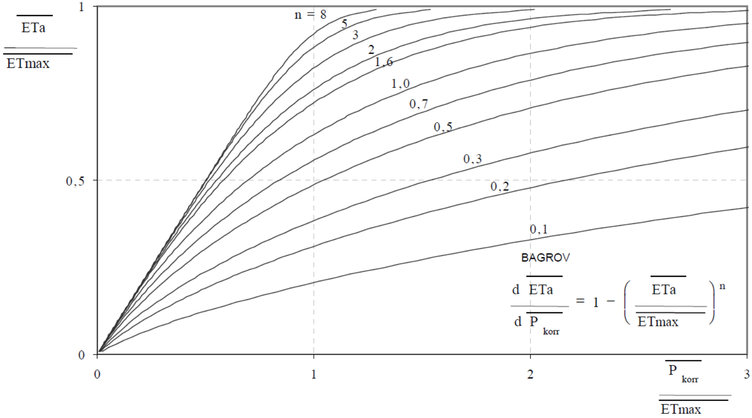
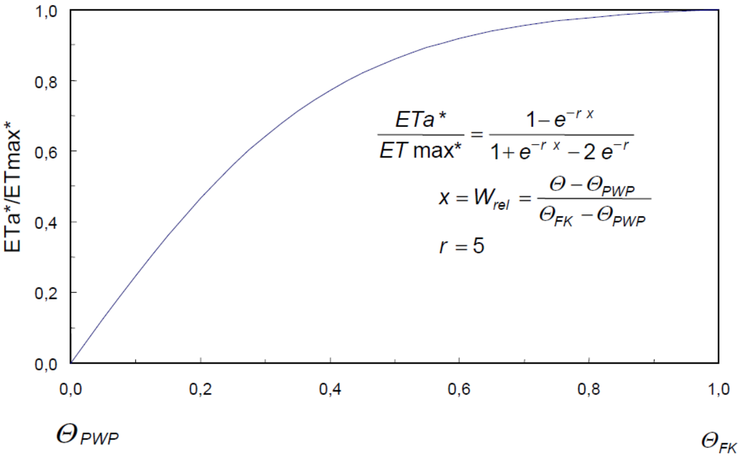
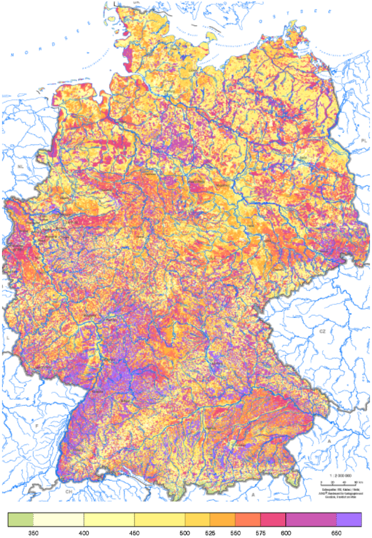


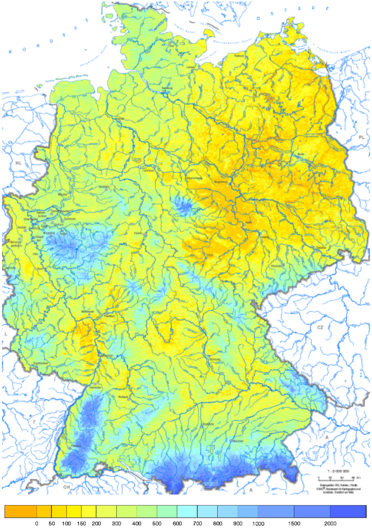
Figure: Reduction of evaporation by (BfG, 2003) [4]

Bagrov-Gugla
Soil Moisture



Runoff in Germany





Further Topics and Readings ...

- Thorntwaite & soil water balance
- Jensen-Haise
- Turc
- Calculation of r_s and r_a
- Wind correction

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