Runoff - Models

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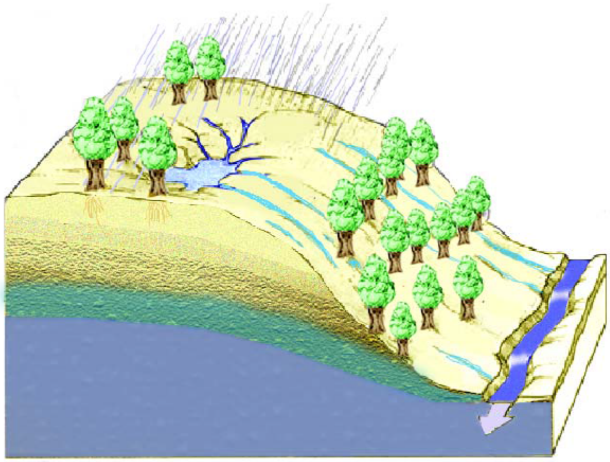
Content

Objectives

Understand Runoff Generation

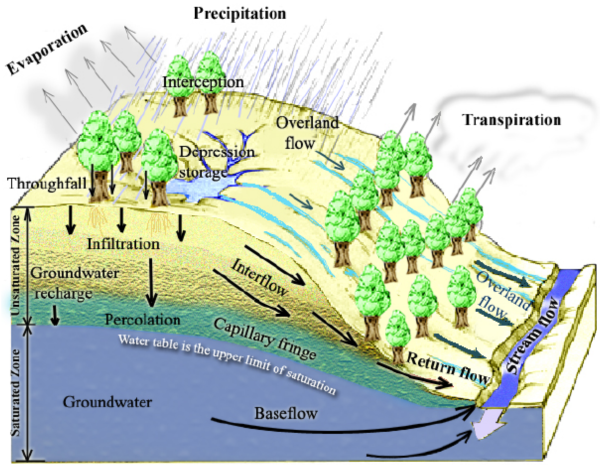
- Runoff production
- Runoff concentration
- Flood routing

Runoff



Tarboton, 2003

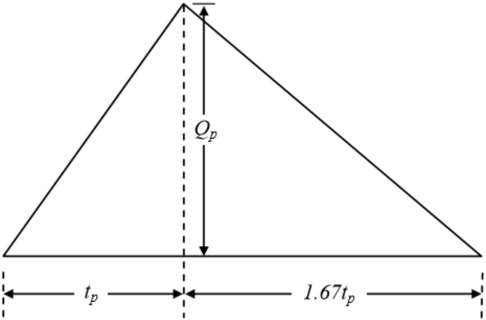
Runoff



Tarboton, 2003

Hydrograph Modeling

Hydrograph
Predicting Elements



NRCS triangular unit hydrograph (SCS, 1985)

Tarboton, 2003

Hydrograph Peak Discharge

$$Q_p = f * C * I * A$$

with	Q_p	Peak discharge	$[m^3/s]$
	f	$f = 0.278 = 1/3.6$	$[-]$
	C	runoff coefficient	$[-]$
	I	precipitation intensity	$[mm/h]$
	A	basin area	$[km^2]$

This method can only be applied in small basins of few km^2 area. It is interesting to note that this method assumes that infiltration is a constant fraction of precipitation.

Coefficients

Runoff coefficients for 2 to 10 year return periods

Description of drainage area	Runoff coefficient
Business	
Downtown	0.70 - 0.95
Neighborhood	0.50 - 0.70
Residential	
Single-family	0.30 - 0.50
Multi-unit detached	0.40 - 0.60
Multi-unit attached	0.60 - 0.75
Suburban	0.25 - 0.40
Apartment dwelling	0.50 - 0.70
Industrial	
Light	0.50 - 0.80
Heavy	0.60 - 0.90
Parks and cemeteries	0.10 - 0.25
Railroad yards	0.20 - 0.35
Unimproved areas	0.10 - 0.30
Pavement	
Asphalt	0.70 - 0.95
Concrete	0.80 - 0.95
Brick	0.75 - 0.85
Roofs	0.75 - 0.95
Lawns	
Sandy soils	
Flat (2%)	0.05 - 0.10
Average (2 – 7 %)	0.10 - 0.15
Steep (≥ 7%)	0.15 - 0.20
Heavy soils	
Flat (2%)	0.13 - 0.17
Average (2 – 7 %)	0.18 - 0.22
Steep (≥ 7%)	0.25 - 0.35

Source: Adapted from ASCE (1992)

Hydrograph
Concentration Time by Kirpich

$$t_c = 0.06625 * \frac{L^{0.77}}{S^{0.385}}$$

with	t_c	concentration time	$[hours]$
	L	Length of basin	$[m]$
	C	Slope	$[-]$

The concentration time is relevant for the selection of the design storm: The design storm should have the duration of the concentration time. Applicable to basins of up to 80 ha. For larger basins apply a correction factor of $f_c = 2 * t_c$ for very small sealed urban basins of $f_c = 0.4 * t_c$.

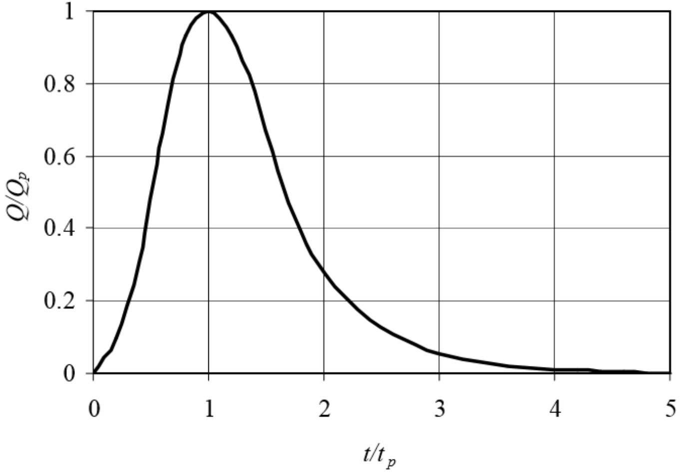
Runoff Amout
Area and Integral of Hydrograph

Runoff Q in [mm] can be calculated using the equation:

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S)}$$

with	F	infiltration amount	$[mm]$
	S	maximum storage	$[mm]$
	Q	runoff	$[mm]$
	P	precipitation	$[mm]$
	I_a	initial loss	$[mm]$

Hydrograph
Total Shape

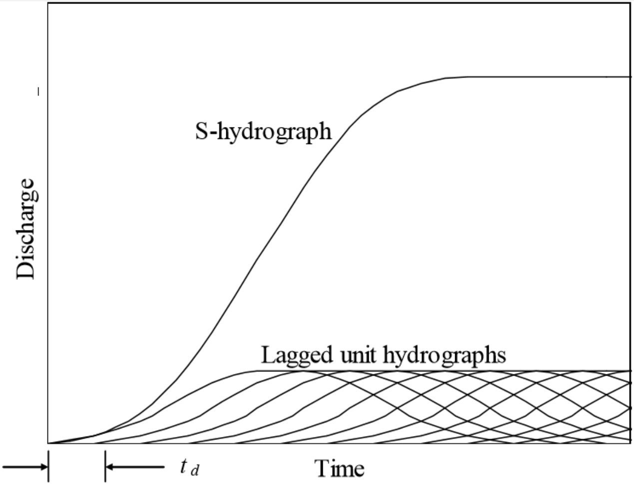


Hydrograph Total Shape

t/t_p	Q/Q_p	t/t_p	Q/Q_p
0.0	0.000	1.4	0.780
0.1	0.030	1.5	0.680
0.2	0.100	1.6	0.560
0.3	0.190	1.8	0.390
0.4	0.310	2.0	0.280
0.5	0.470	2.2	0.207
0.6	0.660	2.4	0.147
0.7	0.820	2.6	0.107
0.8	0.930	2.8	0.077
0.9	0.990	3.0	0.055
1.0	1.000	3.5	0.025
1.1	0.990	4.0	0.011
1.2	0.930	4.5	0.005
1.3	0.860	5.0	0.000

Source: SCS (1985)

S Hydrograph
Superposition of Input Signals

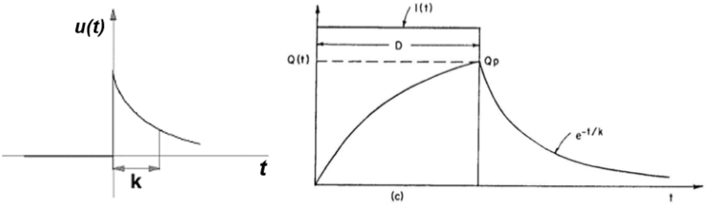


Unit Hydrograph
Convolution

$$
\begin{cases}
Q_1 = P_1 U_1 \\
Q_2 = P_2 U_1 + P_1 U_2 \\
\ldots \\
Q_M = P_M U_1 + P_{M-1} U_2 + \ldots + P_1 U_M \\
Q_{M+1} = 0 + P_M U_2 + \ldots + P_2 U_M + P_1 U_{M+1} \\
\ldots \\
Q_{N-1} = 0 + 0 + \ldots + 0 + 0 + \ldots + P_M U_{N-M} + P_{M-1} U_{N-M+1} \\
Q_N = 0 + 0 + \ldots + 0 + 0 + \ldots + 0 + P_M U_{N-M+1}
\end{cases}
$$

Conceptual Runoff Models

Linear Storage Hydrograph



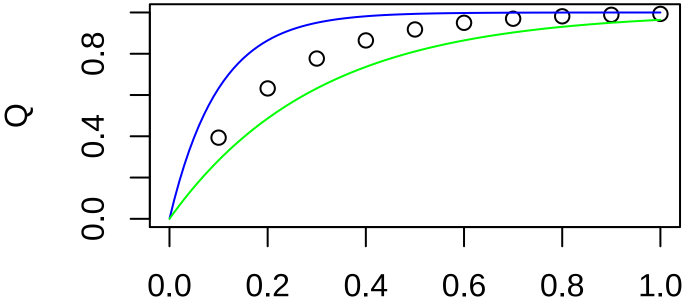
Linear Storage Concept

$$
\begin{aligned}
u_{rl}(t) &= (1/k) * (1 - exp(t/k)) \\
Q_p(t) &= I * (1 - exp(D/k)) \\
u_{fl}(t) &= (1/k) * exp((t - D)/k) \\
Q_{fl}(t) &= Q_p * exp((t - D)/k)
\end{aligned}
$$

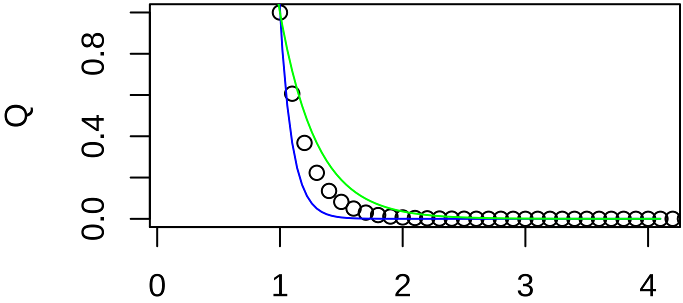
with			
	$u(t)$	unit discharge	$[m^3/s]$
	$Q(t)$	runoff	$[m^3/s]$
	k	recession constant	$[h]$
	t	time	$[h]$
	D	Duration	$[h]$
	I	intensity	$[m^3/h]$

The $u(t)$ unit function describes the reaction of the basin to a unit input, it needs to be multiplied with an input intensity I to yield $Q(t)$. rl, p and fl stand for rising limb, peak and falling limb.

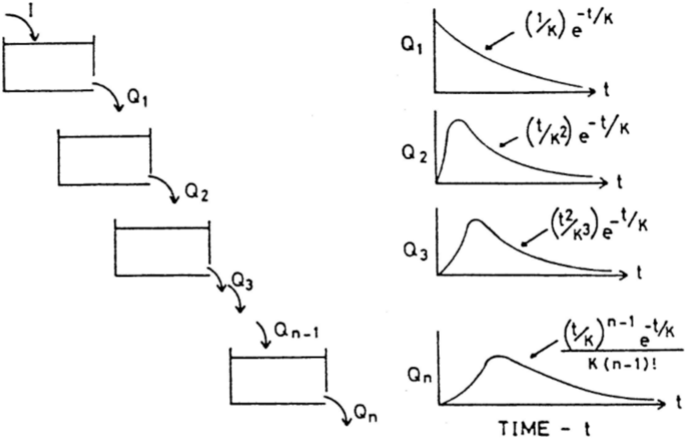
Linear Storage Model
Rising Limb



Linear Storage Model
Recession Limb



n Linear Stores
Nash Cascade



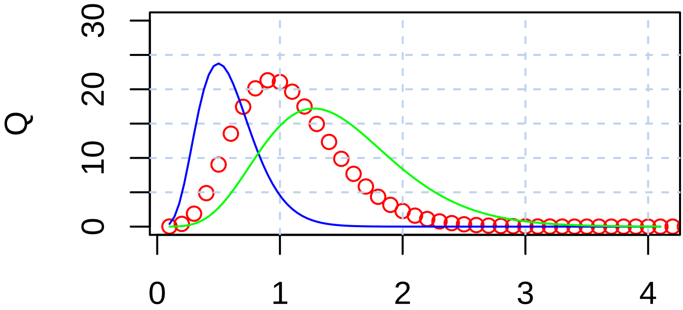
Nash-Cascade
Concept

$$u(t) = \frac{1}{k * \Gamma(n)} * \left(\frac{t}{k}\right)^{(n-1)} * exp\left(-\frac{t}{k}\right)$$

with	$u(t)$	unit discharge	$[m^3/s]$
	k	storage coefficient	$[h]$
	n	number of stores	$[-]$
	t	time	$[h]$
	$\Gamma(n)$	Gamma function of n	$[-]$

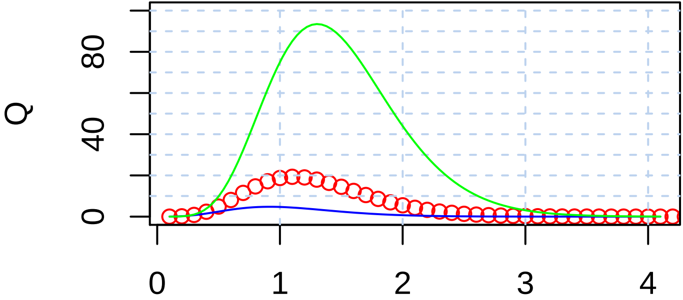
The time t is the time since the onset of rainfall. If n is an integer $\Gamma(n)$ can be replaced by $(n-1)!$.

Nash-Cascade
Changing k



k=0.1 (blue), k=0.3 (green)

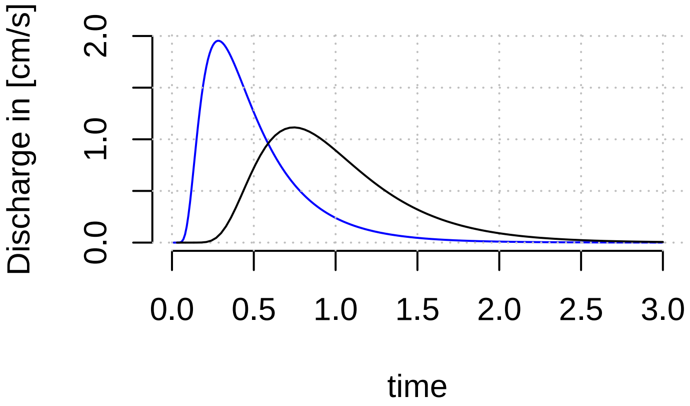
Nash-Cascade
Changing n



n=4 (blue), n=6 (green)

Flood Routing

Diffusion of Flood in Channel
Advection Dispersion Equation



ADE Equation

$$Q_(t) = \frac{x}{\sqrt{4 * pi * D * t^3}} * exp\left[-\frac{(x - v_c * t)^2}{4 * D * t}\right]$$

$$D = v_c * \alpha = v_c * f_D * x$$

with			
	$Q(t)$	discharge	$[m^3/s]$
	x	distance	$[m]$
	D	Dispersion coefficient	$[h]$
	t	time	$[h]$
	v_c	velocity	$[m/s]$
	f_D	scaling factor	$[m]$

Programming in R
ADE

```
c<-1.0
D<-0.1
x1<-0.5
t<-seq(0,3.0,0.01)
u<-x1/(4*pi*D*t^3)^(1/2)*exp(-(x1-c*t)^2/(4*D*t))
plot(t,u)
```

Channel Storage Concept

$$Q_Z - Q_A = \frac{dS}{dt}$$

Q_Z	Inflow to channel
Q_A	Outflow of channel
S	Storage in section
t	time

$$\frac{[Q_Z(t) + Q_Z(t + \Delta t)]}{2} - \frac{[Q_A(t) + Q_A(t + \Delta t)]}{2} = \frac{\Delta S}{\Delta t}$$

Δt	time step
ΔS	change in storage

Storage Change (Continued)

$$\frac{(Q_{Z1}+Q_{Z2})}{2}-\frac{(Q_{A1}+Q_{A2})}{2} = \frac{(S_2-S_1)}{\Delta t}$$
$$\frac{Q_{Z1}}{2}+\frac{Q_{Z2}}{2}-\frac{Q_{A1}}{2}-\frac{Q_{A2}}{2} = \frac{S_2}{\Delta t}-\frac{S_1}{\Delta t}$$
$$\frac{S_2}{\Delta t}+\frac{Q_{A2}}{2}=\left(\frac{S_1}{\Delta t}+\frac{Q_{A1}}{2}\right)-Q_{A1}+\frac{(Q_{Z1}+Q_{Z2})}{2}$$

The equation is solved by substituting the right term for equivalents in the left term of the equation:
$Q_A = f\left(\frac{S}{\Delta t}+\frac{Q_A}{2}\right)$

Muskinghum Method
Storage Difference Equation

The Muskingum-method combines the linear storage term with a non-linear non-steady part for the incoming flood wave:

$$S = K * Q_A + K * X * (Q_Z - Q_A)$$

K is a storage parameter and X is a dimensionless fitting parameter. The equation is written as a difference equation:

$$S = K * (Q_{A2} - Q_{A1}) + K * X * (Q_{Z2} - Q_{Z1} - Q_{A2} + Q_{A1})$$

with $x \leq \frac{\Delta t}{2 * K}$.

Routing Formula
Q_A a Function of Inflow and Storage

The routing formula writes (after re-arranging it):

$$Q_A(t + \Delta t) = C_1 * Q_Z(t + \Delta t) + C_2 * Q_Z(t) + C_3 * Q_A(t)$$

With:

$$\begin{aligned} C_1 &= [-K * X + 0,5 * \Delta t]/[K * (1 - X) + 0,5\Delta t] \\ C_2 &= [+K * X + 0,5 * \Delta t]/[K * (1 - X) + 0,5\Delta t] \\ C_3 &= [K * (1 - X) - 0,5 * \Delta t]/[K * (1 - X) + 0,5\Delta t] \end{aligned}$$

Given: $C_1 + C_2 + C_3 = 1$.

Routing Formula
Simplification and Parametrization

The routing formula is re-written and simplified with coefficients ζ_1 and ζ_2:

$$QA(t+\Delta t) = QA(t)+\zeta_1\,[QZ(t) - QA(t)]+\zeta_2\,[QZ(t+\Delta t) - QZ(t)]$$

With:

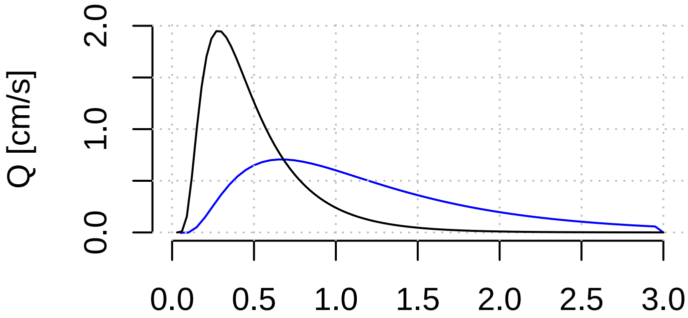
$$\begin{aligned} \zeta_1 = &\quad \Delta t/\,[K(1-X)+0,5*\Delta t] \\ \zeta 2 = &\quad [0,5*\Delta t - K*X]\,/\,[K*(1-X)+0,5*\Delta t] \end{aligned}$$

Parameter X

X has values between 0 and 0.5, so that

$$0 \leq X \leq 0.5$$

If X equals 0.5 there is only translation, with X having a value of 0, there is maximum storage reduction. Most often X has values ranging from 0 to 0.3.

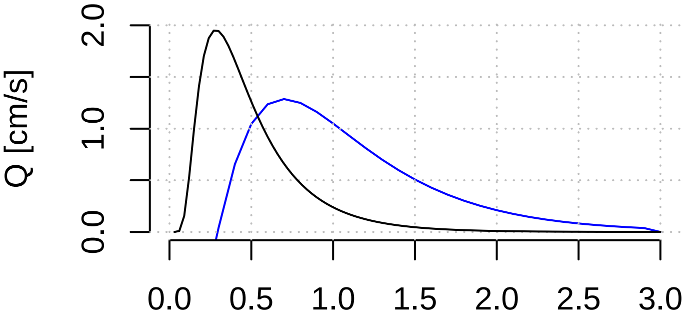


Parameter K

Parameter K has the physical meaning of a residence time.
When chosing K it should be considered and hold that:

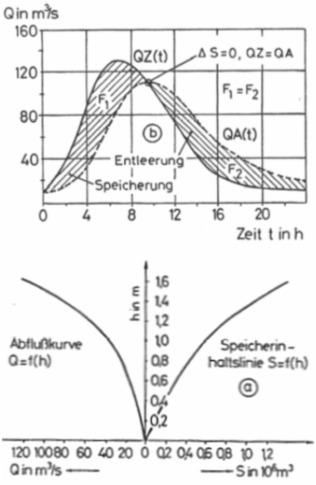
$$2KX \leq dt \leq K$$

In some cases time step needs to be adjusted, here dt=0.1:



```
K<-0.8; X<-0.05 # with 0 < X < 0.5
muskinghum <- function(QA1,QZ1,QZ2,K,X){
   c1  <- dt/(K*(1-X)+0.5*dt)
   c2  <- (0.5*dt-K*X)/(K*(1-X)+0.5*dt)
   v <-  QA1 + c1* (QZ1 - QA1) + c2*(QZ2 - QZ1)
   return(v) # return of QA2}
l  <- length(u)
dt <- (tmax - t0)/(l-1)
QA2<- array(0.0, dim=c(l))
i  <- 1
while (i < l) {
   QZ1 <- u[i]; QZ2 <- u[i+1]
   if (i <= 2) {QA1 <- QZ1} # critical
   QA2[i] <- muskinghum(QA1,QZ1,QZ2,K,X)
   QA1 <- QA2[i]
   i <- i + 1 }
```

Lake Storage



- Overlay of QZ(t) and Discharge QA(t)
- Fill (F1) and Empty (F2) Storage (S)
- $\Delta S = 0$ when QA(t)=QZ(t)
- Discharge curve $Q_A = f(h)$
- Storage volume $S = f(h)$

Quelle: Maniak (2010)

Summary

- Point infiltration: effective Parameters
- Basin infiltration
- Constant rate loss: Mulvaney
- Conceptual Models
- Hydrological routing