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Sustainable Water Resources Management

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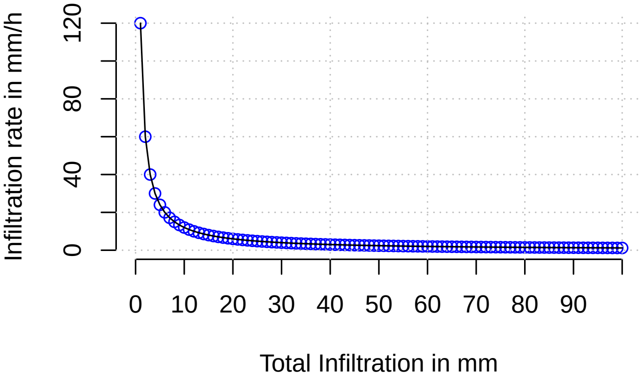
Outline

Infiltration of ponded areas can be calculated using the ponding depth and suction at the wetting front [1].

$$\frac{F}{dt} = K * \left[1 + \frac{(\theta - \theta_0) * (\psi_f + h_p)}{F}\right] \tag{1}$$

F total infiltration in [mm], t time, K hydraulic conductivity [L/T], θ moisture content of soil $\theta_0 =$ residual water content before infiltration, $\psi_f =$ suction at the wetting front [L] and $h_p =$ ponding depth [L].


Infiltration rate in mm/h
0
40
80
120
0
10
20
30
40
50
60
70
80
90
Total Infiltration in mm

Pores

- **Macro pores** with a diameter $> 50\mu m$, for water movement, aeration, no holding capacity
- **Meso pores**, diameter from 0,2 to 10 μm, hold water against gravity
- **Fine pores**, diameter $<$ 0,2 μm, not accessible for plants, dead water.

- **Primary pores** deposition and sedimentation, voids in sediment
- **Secondary pores**, cracks, fissures, roots, fractures

Porosity

$$n_e = \frac{V_p}{V_{gesamt}} \tag{2}$$

with n_e porosity, V_p pore volume and V_{gesamt} total soil volume.

- For water movement effective porosity counts
- at saturation $V_p = V$ total water volume
- water with density $\rho = 1g/cm^3$, hence m_w/V=V_w/V

Capillary Rise

Capillary rise results from interaction of liquid with solid phase.

Capillary rise amounts to:

$$h = \frac{2\sigma \cos\theta}{\rho g r} \tag{3}$$

with σ surface tension in J/m^2, θ contact angle, ρ density of liquid in g/cm^3, g gravitational acceleration kg/m^3 and r radius of pore or pipe in m. Capillary rise therefore amounts to about:

$$h \approx \frac{1.4 * 10^{-5} m^2}{r} \tag{4}$$

Table 1: Capillary rise

Radius of capillary	capillary rise
1 cm	1.4 mm
1 mm	14 mm
0.1 mm	14 cm
0.01 mm	1.4 m

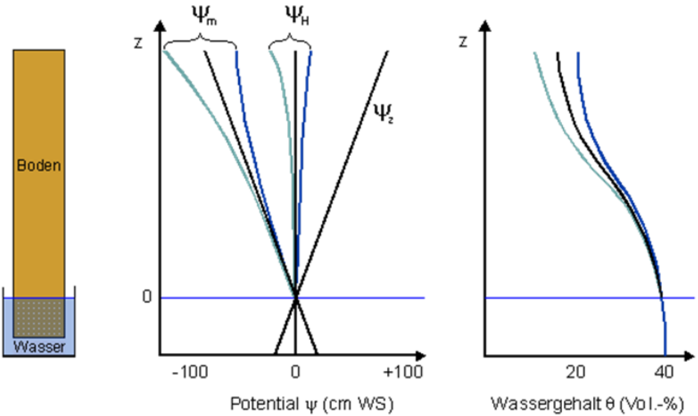


Figure 1: Potential theory

Potential theory

- Soil water movement is based on potential theory.
- Energy potential is determined.
- Water movement always to lowest potential.
- Symbol for potential is Ψ

$$\Psi = m * g * h \tag{5}$$

with Ψ representing potential, m Mass in kg, g gravitational accelaration 9.81 m/s^2 and h height above reference level in m.

Potentials in Soils

1. All potentials in soil are measured, calculated or determined in any way:
   - Hydrostatic potential Ψ_h
   - Matrix potential Ψ_m
   - Osmotic potential Ψ_o for saline soils
2. Then total potential Ψ is determined.
3. Gradient per unit length z is calculated.
4. Movement towards lowest potential.

Potential Equation

$$\Psi = \Psi_h + \Psi_m + \Psi_o \tag{6}$$

with: Hydrostatic potential Ψ_h, Matrix potential Ψ_m, osmotic potential Ψ_o for salty soils

Potentials are given as pressure in hPa or as height, corresponding to work required to lift water to level h Ψ_h or against suction Psi_m or against osmotic pressure.

pF value

The logarithm of the head in [cm] corresponding to the negative pressure produced by capillary forces represents the pF value

- pF $=$ 2 corresponds to negative pressure of 100 cm
- pF $=$ 4 corresponds to 10.000 cm
- pF $=$ 0 corresponds to saturated conditions.

Field capacity

Within 1-2 days after rainfall or irrigation water movement and seepage seems to stop. Flow velocities are very small. The amount of water held against gravity by the soil after 1-2 days is the **field capacity (FC)**.

(Schachtschabel et al. 1998)

Determining the field capacity

- Saturate soil and cover (field), saturated soil cylindre
- Measure moisture after 2 days
- Definition: Field capacity is moisture content at pF $= 1.8$

Permanent wilting point - PWP

- Plants start wilting at pF-Wert $= 4.2$
- This corresponds to more than 10.000 cm water column.
- Plants cannot produce a suction larger than this capillary force.

Effective field capacity - efc

The effective field capacity corresponds to FK-PWP

$$nFK = FK - PWP \tag{7}$$

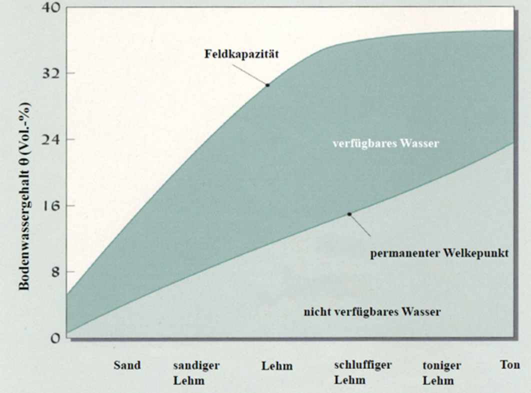


Figure 2: Effective field capacity and texture

Einteilung der Poren und Kennwerte des Bodenwasserhaushaltes				
Kategorie	Ausprägungen			
Porenbereiche	weite Grobporen	enge Grobporen	Mittelporen	Feinporen
Saugspannung in hPa	unter 60	60 bis 300	300 bis 15000	über 15000
pF-Wert	unter 1,8	1,8 bis 2,5	2,5 bis 4,2	über 4,2
Äquivalentdurchmesser in µm	über 50	50 bis 10	10 bis 0,2	unter 0,2
Funktion der Poren	schnell bewegliches	langsam bewegliches	pflanzen-verfügbares	nicht pflanzen-verfügbares
	Sickerwasser		Haftwasser	
Kennwerte Kurzzeichen	Luftkapazität LK	nutzbare Feldkapazität nFK		Totwasser TOT

Figure 3: Pore size and water holding capacity

Richards Equation

$$v_{us} = k(\theta(\psi)) * d\Omega = k(\theta(\psi)) * \frac{d(h+\psi)}{dz} \quad (8)$$

unsaturated hydraulic conductivity v_{us} in [m/s],
soil moisture θ [-],
matrix potential ψ in [m] expr. as water head,
potential Ω in [m], and ponding head h in [m],
soil depth z in [m].

Buckingham Equation

$$v_{us} = k(\theta) * \frac{d(\theta)}{dz} \tag{9}$$

v_{us} function of soil moisture θ,
$k(\theta)$ function of soil moisture

Disadvantage

- No modeling of positive pressure (ponding)
- Boundary conditions limited

Effective Saturation S

$$S = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{10}$$

saturationθ reduced by residual water content θ_r
normalized by effective moisture range $\theta_s - \theta_r$.

The saturation can be expressed as a function of matrix potential $S(\psi)$:

$$S = [1 + (\alpha * |\psi|)^n]^m \tag{11}$$

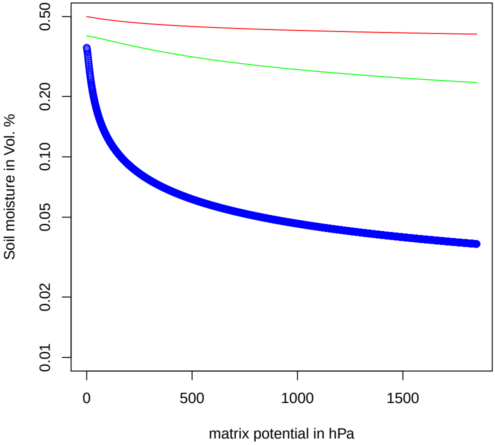
Re-arranging we obtain the Van-Genuchten equation producing soil moisture as a function of suction head or matrix potential:

$$\theta(\psi) = \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha * |\psi|)^n]^m} \tag{12}$$

with $\theta(psi)$ soil moisture in [Vol. $\%$], θ_s soil moisture at saturation [Vol. $\%$], θ_r residual moisture at wilting point in [Vol. $\%$] and α, n and m are parameters, ψ matrix potential in [hPa]. Parameter m can be derived from n by $m = 1 - 1/n$.

Table 2: Parameters for Van Genuchten equation

Soil type	θ_r	θ_s	α	n
Ss	**0.043019**	**0.370687**	**0.087424**	**1.57535**
fS	0.005931	0.400616	0.050887	1.46409
Tl	0.003932	0.501398	0.033118	1.06283



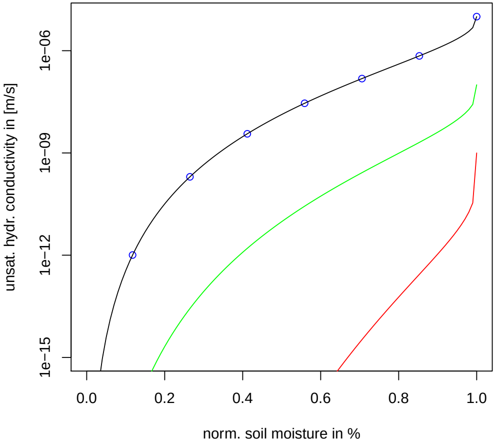
Soil moisture in Vol. %
0.50
0.20
0.10
0.05
0.02
0.01
0
500
1000
1500
matrix potential in hPa

A relationship between the unsaturated hydraulic conductivity $K(\theta)$ and saturation S can be derived from the Van-Genuchten relationship:

$$K(\theta) = K_s * S^l * \left[1 - \left(1 - S^{1/m}\right)^m\right]^2 \tag{13}$$

This function requires parameters soil moisture θ, soil specific parameter n to derive $m = 1 - 1/n$. Values for n are given as a function of soil texture, organic matter content etc. l is assumed to be 0.5, residual moisture θ_r and max soil moisture θ_s are given in tables or can be derived empirically.

The unsaturated hydraulic conductivity can be calculated based on the Van-Genuchten equation [2]. Blue for Sand (Ss) with $n=1.5$, green for silt (Uu) with $n=1.3$ and red for clay (Tt) with $n=1.1$.


unsat. hydr. conductivity in [m/s]
1e−06
1e−09
1e−12
1e−15
0.0
0.2
0.4
0.6
0.8
1.0
norm. soil moisture in %

With pedotransfer functions parameters for the Van Genuchten equation or other soil physical parameters can be derived from soil properties such as soil type, texture, organic matter content etc.

- organic matter
- clay [%]
- sand [%]
- density $[g/cm^3]$

References

[1] J. R. Philip. An infiltration equation with physical significance. *Soil Sci.*, 77:153157, 1954.

[2] M. Th. Van Genuchten. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Sci. Soc. Am. J.*, (44):892898, 1980.