FACH HOCHSCHULE LÜBECK

Infiltration and Runoff

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Content

Objectives

Understand Infiltration Processes

- Fundamental soil physics
- Know most important formulae
- Know measurement techniques
- Understand relevance for hydrology
- Be able to apply infiltration models

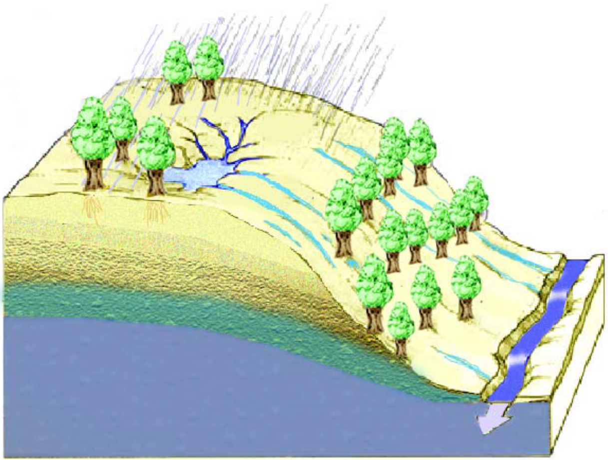
Infiltration
Switch in Flowpaths

Infiltration is an important switch in hydrological flow paths, dividing precipitation into surface flows and subsurface flows.

$$Q_s = P - I \, [\mathrm{mm}]$$

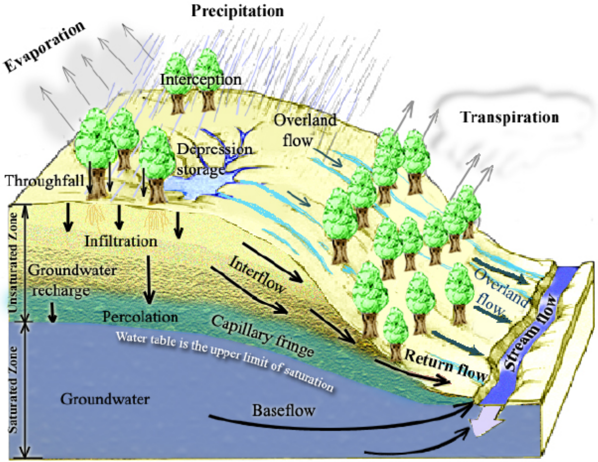
mit Q_s = surface runoff in mm, P Precipitation, I Infiltration in mm.

Infiltration and runoff



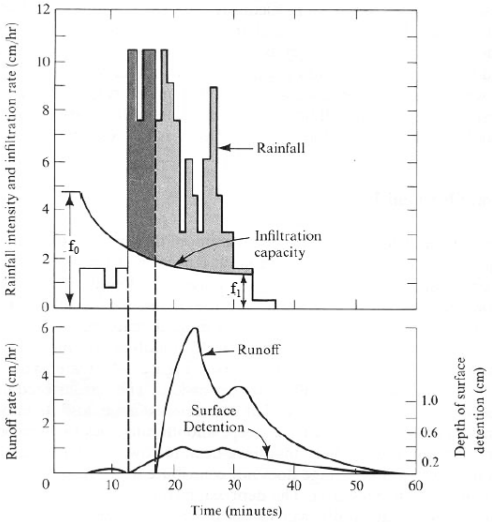
Tarboton, 2003

Infiltration and runoff



Tarboton, 2003

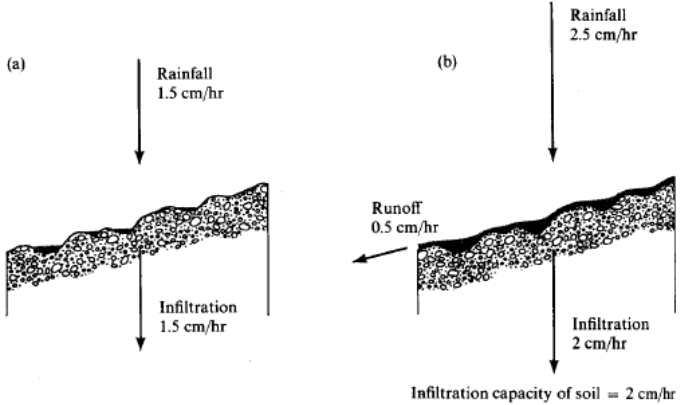
Infiltration and runoff



Dunne & Leopold, 1997

- Infiltration rate decreases
- Approaches constant value

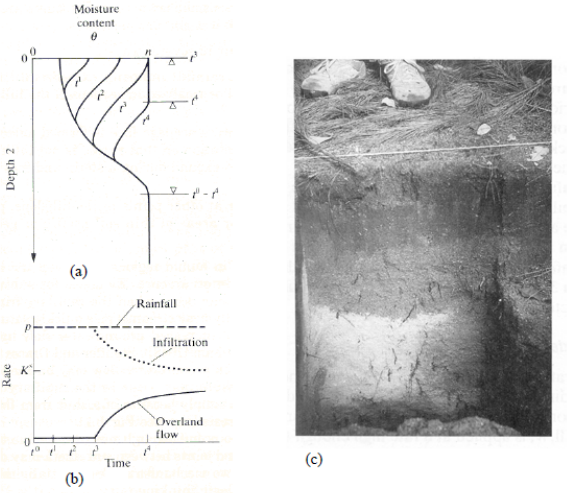
Infiltration and runoff



Dunne & Leopold, 1997

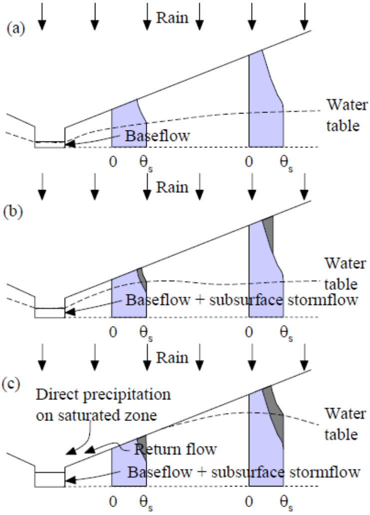
- $P > I$ infiltration and runoff
- $P <= I$ infiltration, no runoff

Infiltration and runoff



Bras, cited in Tarboton, 2003

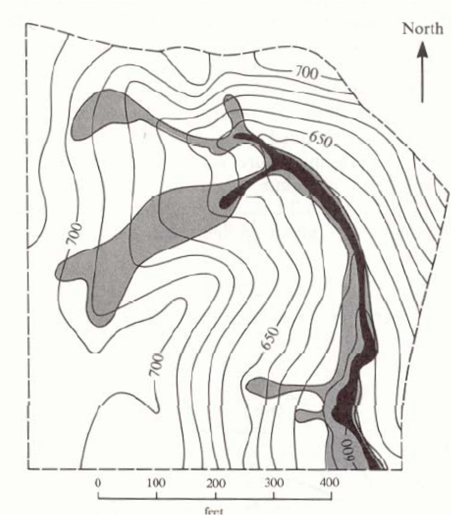
Infiltration and runoff



Tarboton, 2003

- Infiltration saturates soil
- Saturated soil produces runoff
- Floodplain turns into runoff producing area

Infiltration and runoff



Dunne & Leopold, 1997

- Saturated area near rivers
- Runoff producing areas
- High correlation of moisture with runoff coefficient

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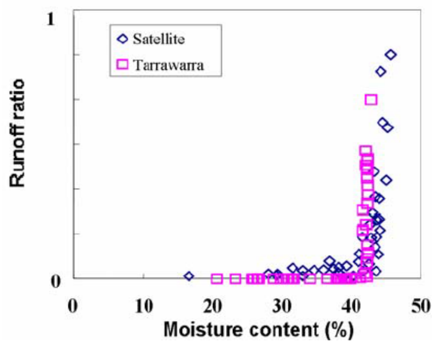
Infiltration and runoff



Relationship between runoff ratio and soil moisture content (Woods et al., 2001, Copyright, 2001, American Geophysical Union, reproduced by permission of American Geophysical Union).

Tarboton, 2003

Transmission losses

Flood wave advancing over a dry streambed in Walnut Gulch experimental watershed where channel transmission losses are considerable (Courtesy of David Goodrich, USDA-ARS).

Goodrich, USDA cited in Tarboton, 2003

Infiltration models

Philip Infiltration Formula
Infiltration Amount

Philip wrote a famous paper on the theory of infiltration in 1969: Philip J R. Theory of infiltration. Advan. Hydrosci. 5:215-96, 1969. [Division of Plant Industry, CSIRO, Canberra, Australia]

$$I = S * \sqrt{t} + a * t$$

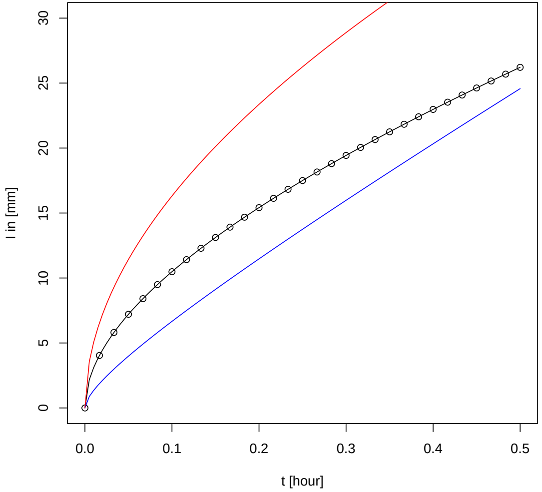
with	I	infiltration amount	$[mm]$
	S	sorptivity	$[mm/h^{-0.5}]$
	a	hydraulic conductivity	$[mm/h]$
	t	time	$[hours]$

R program

The infiltration formula can be plotted using an R program.

```
S <- 30 # mm
a <- 8  # mm/hour
t <- seq(0,0.5,by=1/60.0)
I <- S*t^(0.5)+a*t
plot(t,I)
```

R program of infiltration amount (Philip)



- example with S=30 mm, a=10 m/hour
- low, infiltration rate, high sorption (red)
- high infiltration rate, low sorption (blue)

Philip Infiltration Formula
Infiltration rate

The infiltration rate *i* in *mm* per time can be derived from the derivation of the Philip formula for infiltration amount (or vice vs. amount by integration of rate):

$$i = \frac{1}{2} * S * t^{-0.5} + a$$

with	i	infiltration rate	$[mm/h]$
	S	sorptivity	$[mm/h^{-0.5}]$
	a	hydraulic conductivity	$[mm/h]$
	t	time	$[h]$

R program

The infiltration rate formula can be plotted using an R program.

```
S <- 30 # mm
a <- 8  # mm/hour
t <- seq(0,1,by=1/60.0)
i <- 0.5*S*t^(-0.5) + a
plot(t,i)
```

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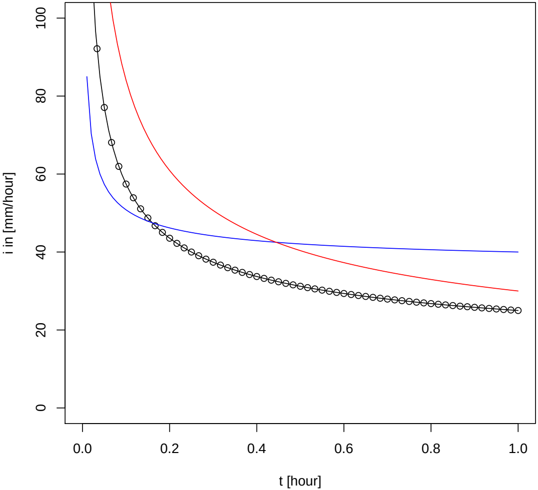
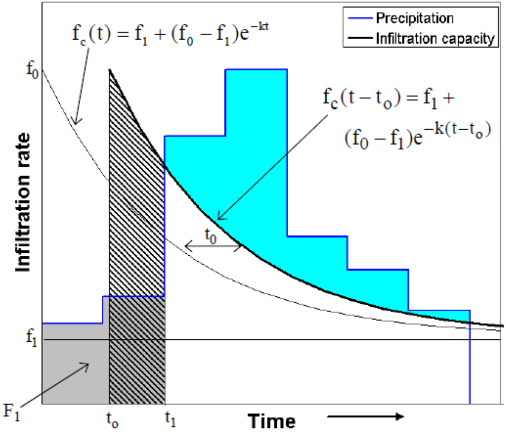
R program of infiltration rate (Philip)



- example with S=30 mm, a=10 m/hour
- low, infiltration rate, high sorption (red)
- high infiltration rate, low sorption (blue)

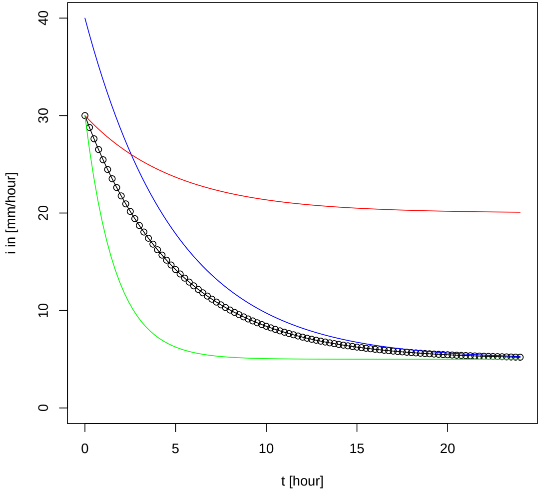
Horton equation



cited in Tarboton, 2003

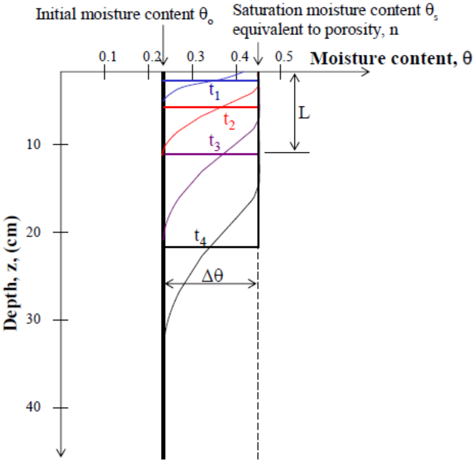
- f_1 constant infiltration rate
- f_0 initial infiltration rate
- k recession factor

R program of Horton equation



- example with $f_0 = 30$ mm, $f_1 = 5$ mm/hour
- higher initial rate (blue)
- higher final rate (red)
- higher recession constant (green)

Green & Ampt equation



- Assuming block sharp wetting front
- Analytical solution
- Can include ponded water

cited in Tarboton, 2003

Green & Ampt equation

At any time, t, the penetration of the infiltrating wetting front will be Z. Darcy's law can be stated as follows:

$$q = \frac{dI}{dt} = -K_s * \left[\frac{h_f - (h_s + Z)}{Z} \right]$$

where K_s is the hydraulic conductivity corresponding to the surface water content, and $I(t)$ is the cumulative infiltration at time t, and is equal to $Z * (\theta_s - \theta_0)$.

Green & Ampt equation

Using this relation for $I(t)$ to eliminate Z and performing the integration yields,

$$I = K_s * t - (h_f - h_s) * (\theta_s - \theta_0) * log_e \left(1 - \frac{I}{(h_f - h_s) * (\theta_s - \theta_0)}\right)$$

with	$I(t)$	infiltration amount	$[cm]$
	K_s	hydr. conductivity	$[cm/h]$
	h_f	wetting front pressure head (negative)	cm
	h_s	water pressure at surface (ponding)	cm
	θ_s	moisture content at saturation	$-$
	θ_0	antecedent moisture	$-$

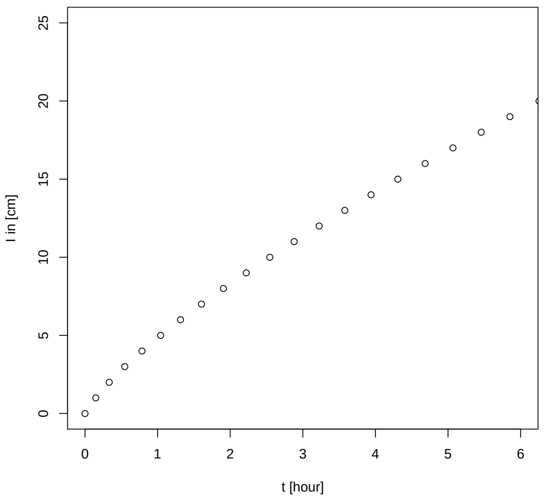
Green & Ampt equation

In order to solve this equation, we need to bring $I(t)$ to one side of the equation:

$$\frac{1}{K_s} * \left[I - (h_f - h_s) * (\theta_s - \theta_0) * log_e \left(1 - \frac{I}{(h_f - h_s) * (\theta_s - \theta_0)} \right) \right] = t$$

with	$I(t)$	infiltration amount	$[cm]$
	K_s	hydr. conductivity	$[cm/h]$
	h_f	wetting front pressure head (negative)	cm
	h_s	water pressure at surface (ponding)	cm
	θ_s	moisture content at saturation	$-$
	θ_0	antecedent moisture	$-$

R program of Green & Ampt equation



- implicit: insert *I* to get *t*
- plot *t* (result) versus *I* (input)
- works though!
- calculates impact of ponding

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Green & Ampt parameters

Green – Ampt infiltration parameters for various soil classes (Rawls et al., 1983). The numbers in parentheses are one standard deviation around the parameter value given.

Soil Texture	Porosity n	Effective porosity θ_e	Wetting front soil suction head $\lvert\psi_f\rvert$ (cm)	Hydraulic conductivity K_{sat} (cm/hr)
Sand	0.437 (0.374-0.500)	0.417 (0.354-0.480)	4.95 (0.97-25.36)	11.78
Loamy sand	0.437 (0.363-0.506)	0.401 (0.329-0.473)	6.13 (1.35-27.94)	2.99
Sandy loam	0.453 (0.351-0.555)	0.412 (0.283-0.541)	11.01 (2.67-45.47)	1.09
Loam	0.463 (0.375-0.551)	0.434 (0.334-0.534)	8.89 (1.33-59.38)	0.34
Silt loam	0.501 (0.420-0.582)	0.486 (0.394-0.578)	16.68 (2.92-95.39)	0.65
Sandy clay loam	0.398 (0.332-0.464)	0.330 (0.235-0.425)	21.85 (4.42-108.0)	0.15
Clay loam	0.464 (0.409-0.519)	0.309 (0.279-0.501)	20.88 (4.79-91.10)	0.1
Silty clay loam	0.471 (0.418-0.524)	0.432 (0.347-0.517)	27.30 (5.67-131.50)	0.1
Sandy clay	0.430 (0.370-0.490)	0.321 (0.207-0.435)	23.90 (4.08-140.2)	0.06
Silty clay	0.479 (0.425-0.533)	0.423 (0.334-0.512)	29.22 (6.13-139.4)	0.05
Clay	0.475 (0.427-0.523)	0.385 (0.269-0.501)	31.63 (6.39-156.5)	0.03

- K_{sat} corresponds to K_s
- $|\psi_f|$ corresponds to h_f
- θ_e corresponds to θ_s
- θ_0 needs to be known

Summary

- Measurement methods: Guelph, double ring and suction plate
- Models: Philip, Green-Ampt, Haverkamp
- Application: Runoff modeling, recharge, pollutants