Tracers in Hydrology

Modeling

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Content

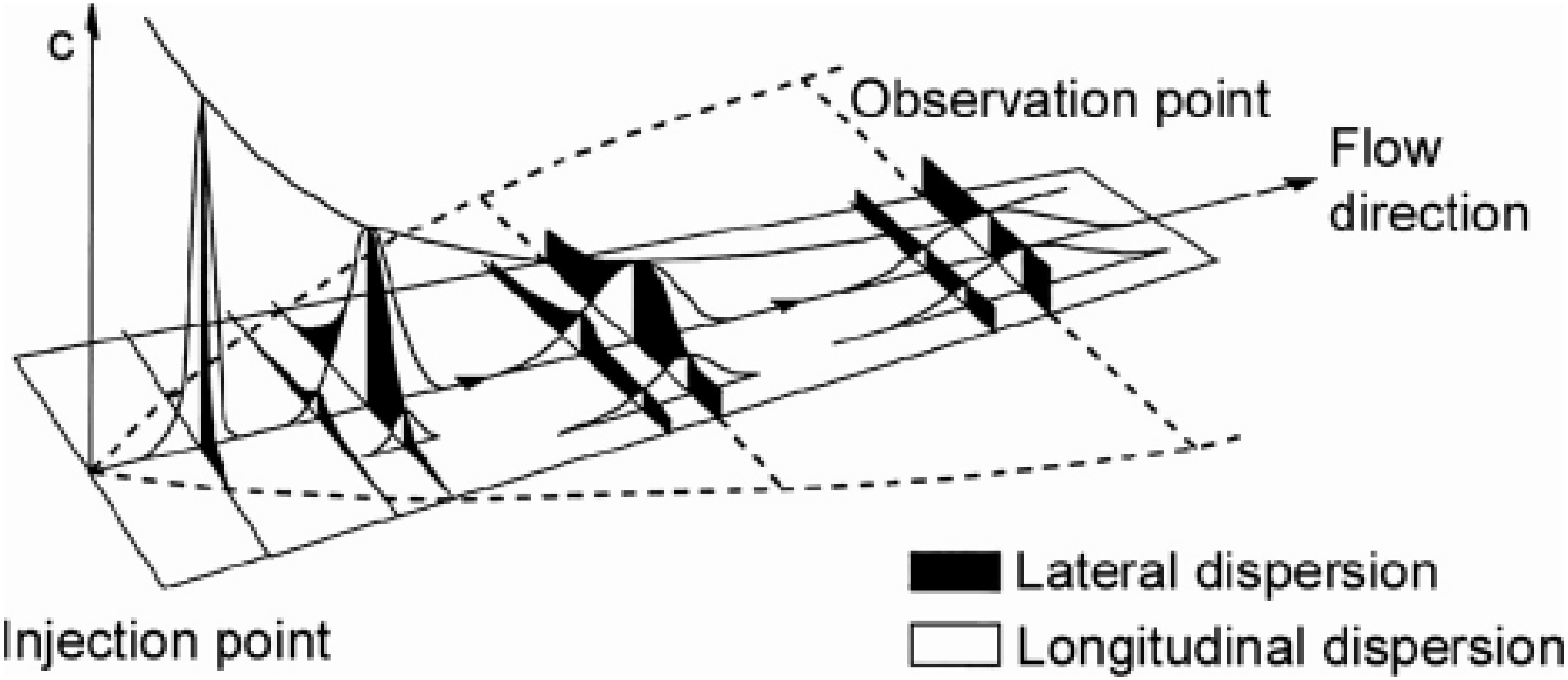
Transport models

- Conceptual models:
  - Compartment models
  - Application
  - Limits
- Physical models
  - *Transport equation*
  - *Solutions:*
    - *analytical*
    - *numerical*

Rhine Alarm Model

Basics and Theory of Solute Transport

Longitudinal and Transversal Dispersion



- Dispersion transveral and longitudinal
- Gauß-shaped curves

Longitudinal and Transversal Dispersion

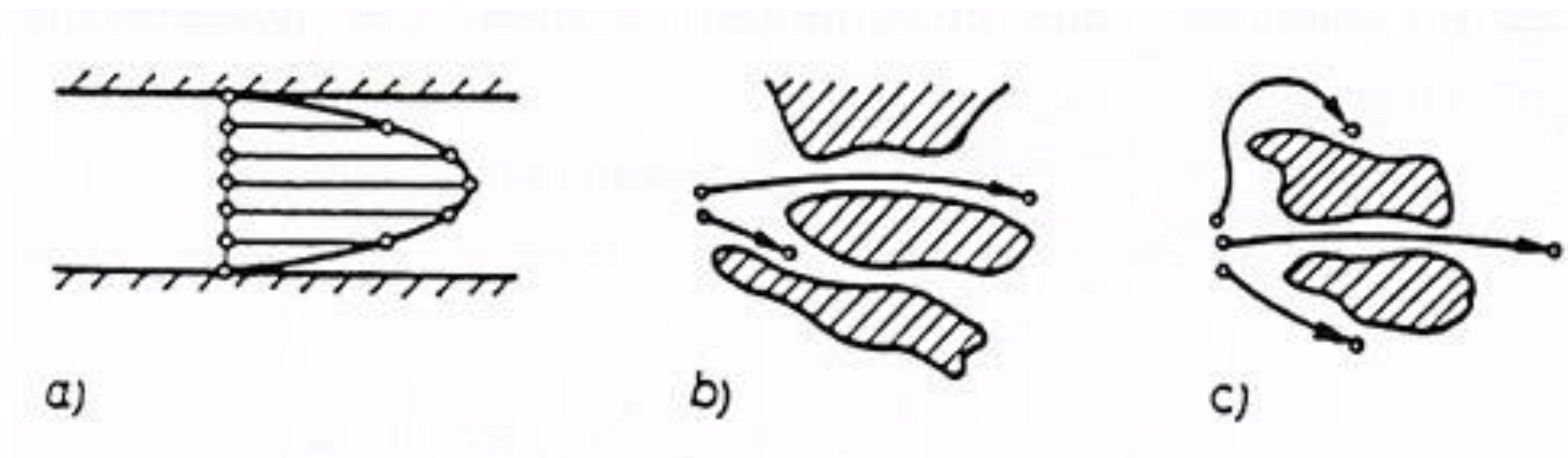
Dispersion of solutes during transport

Concentration decreases and an increasingly large volume of water is contaminated

a) Different volecities

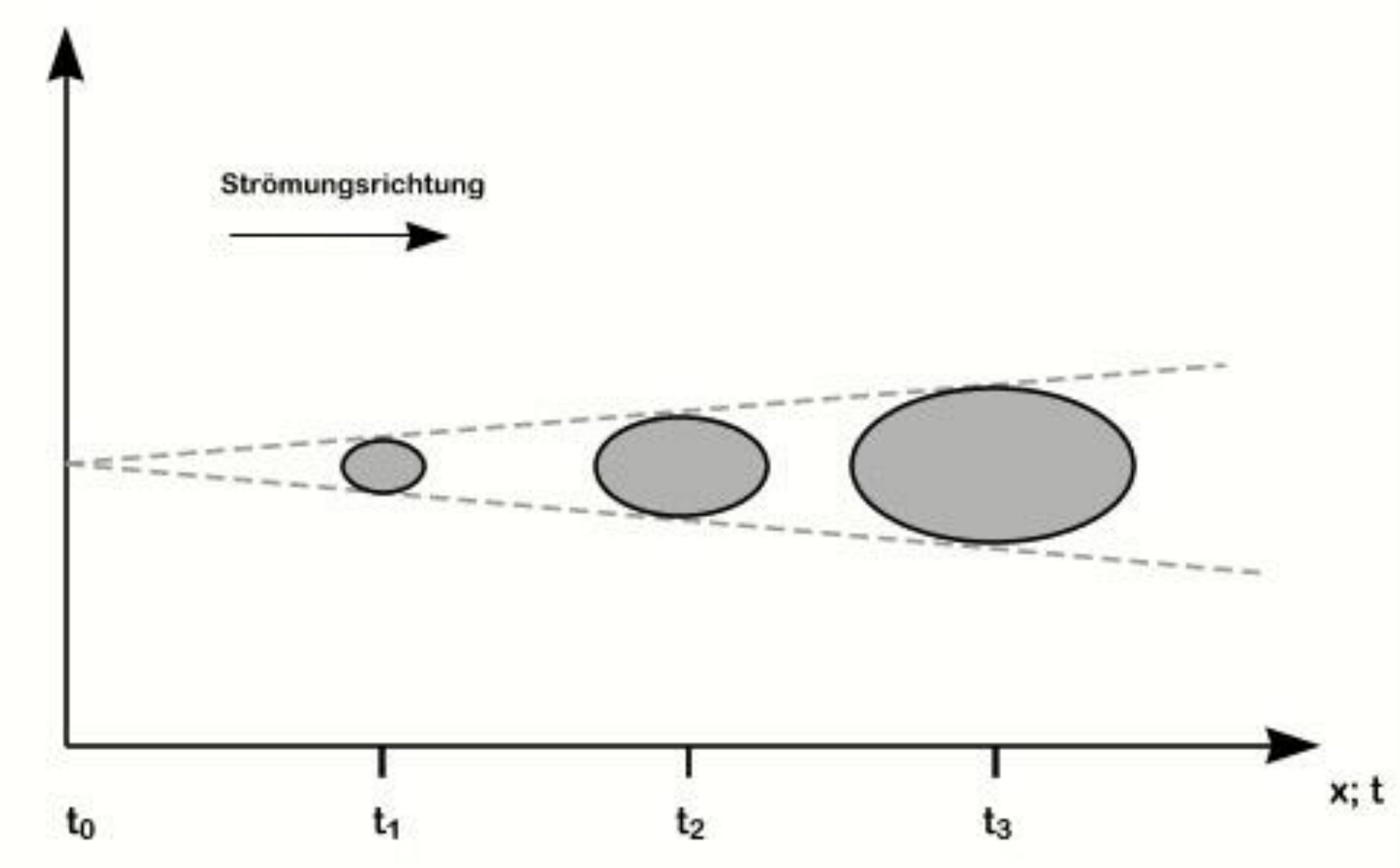
b) Different pore size

c) Different paths

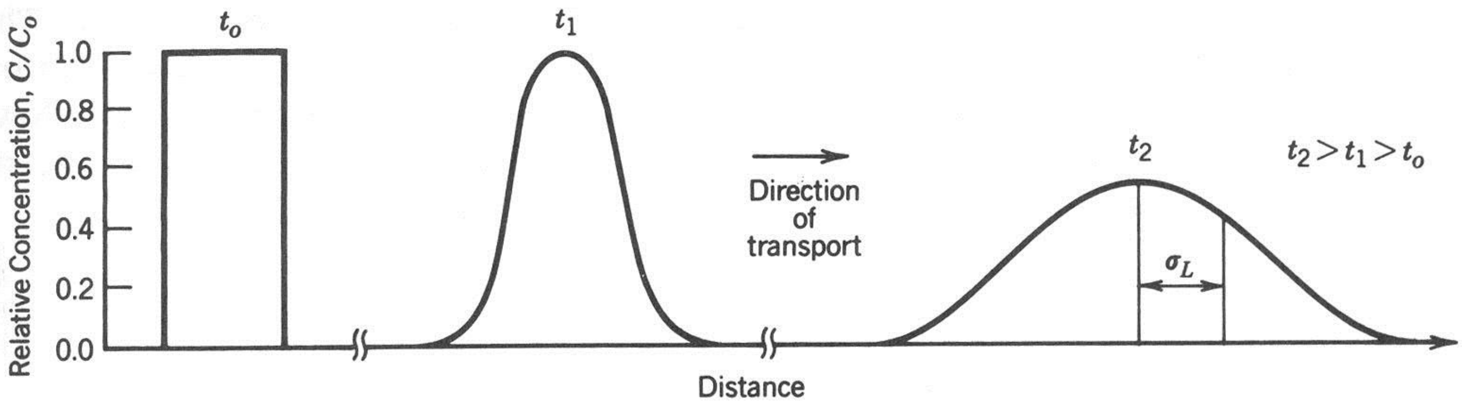


Quelle: Sächsisches Landesamt für Umwelt und Geologie

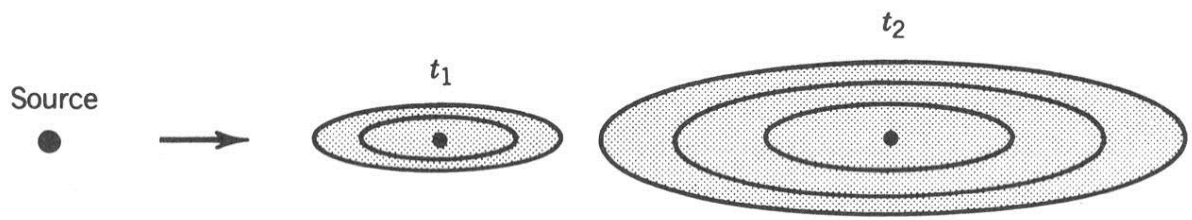
Effect of hydrodynamic dispersion



Quelle: Sächsisches Landesamt für Umwelt und Geologie



(a)



(b)

Figure 10.7

Variation in concentration of a tracer spreading in (*a*) one or (*b*) two dimensions in a constant velocity flow system.

Dispersion coefficient

- Dispersion coefficient D (m^2/s)

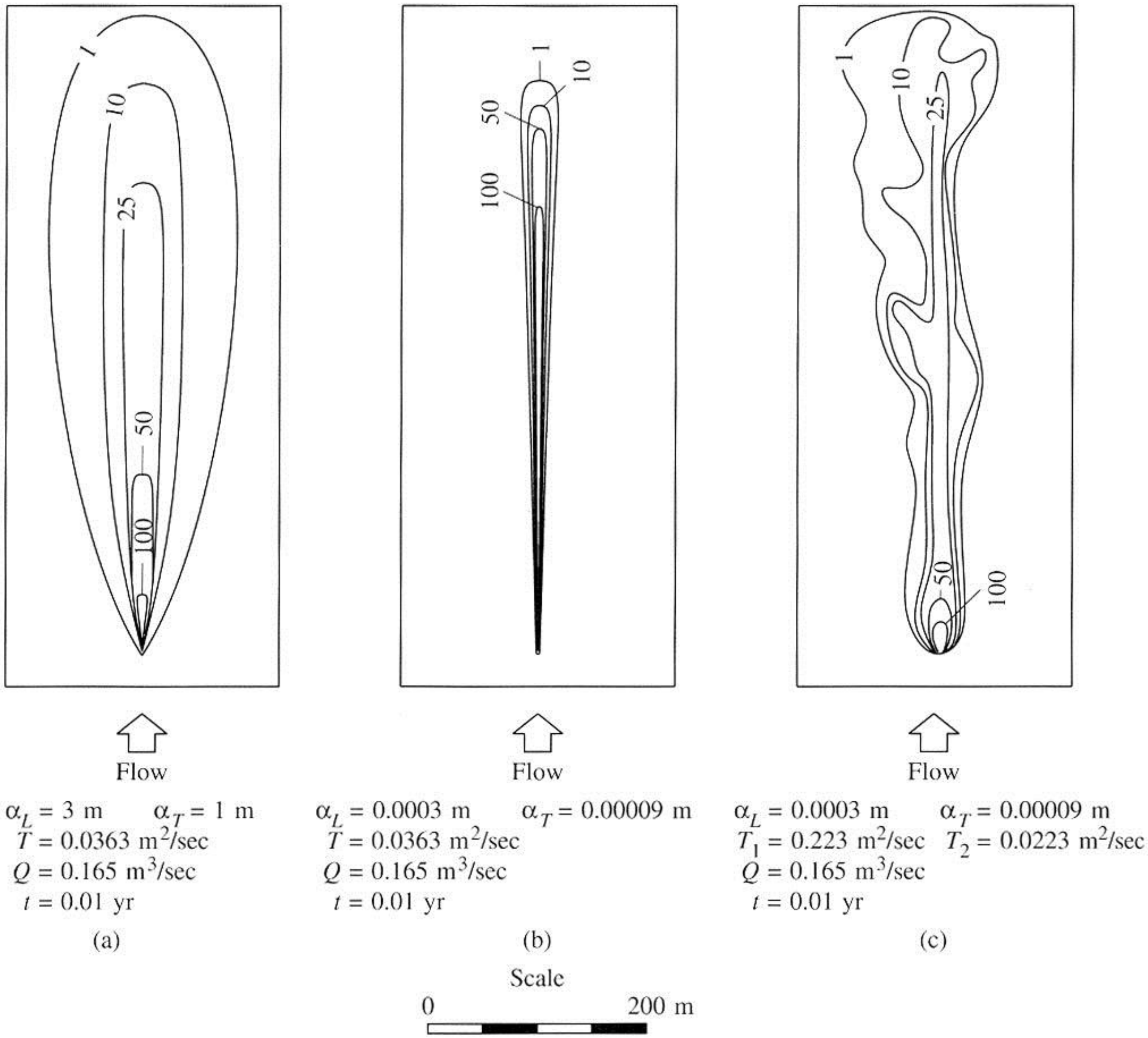
  $D = \delta * v_a$

  dispersivity δ is scale dependent and describes the heterogeneity
- Laboratory scale
- Field scale
- Regional scale

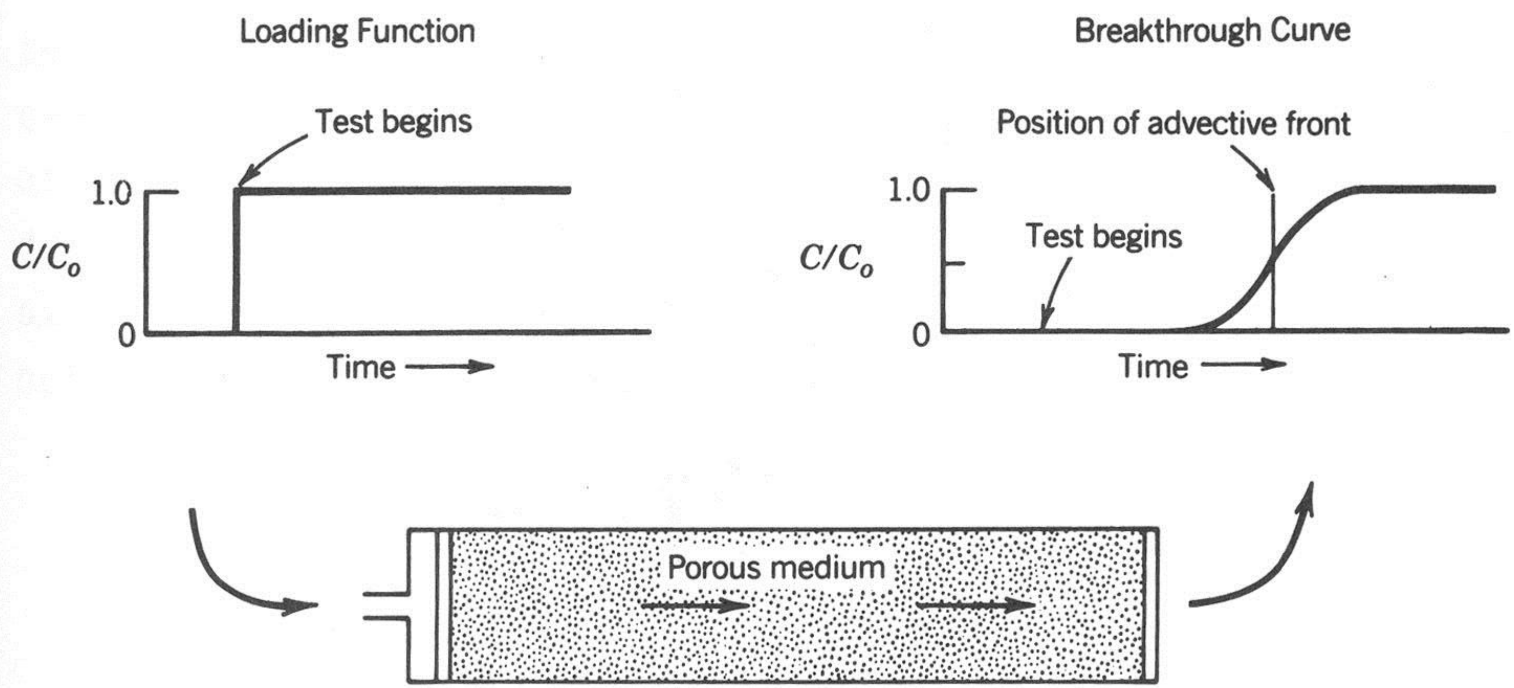
Transversal / longitudinal Dispersion

- Transversal dispersion 0,1- to 0,2-times longitudinal
- Hydrodynamic dispersion can be determined with column or field experiments

Large and small dispersivity



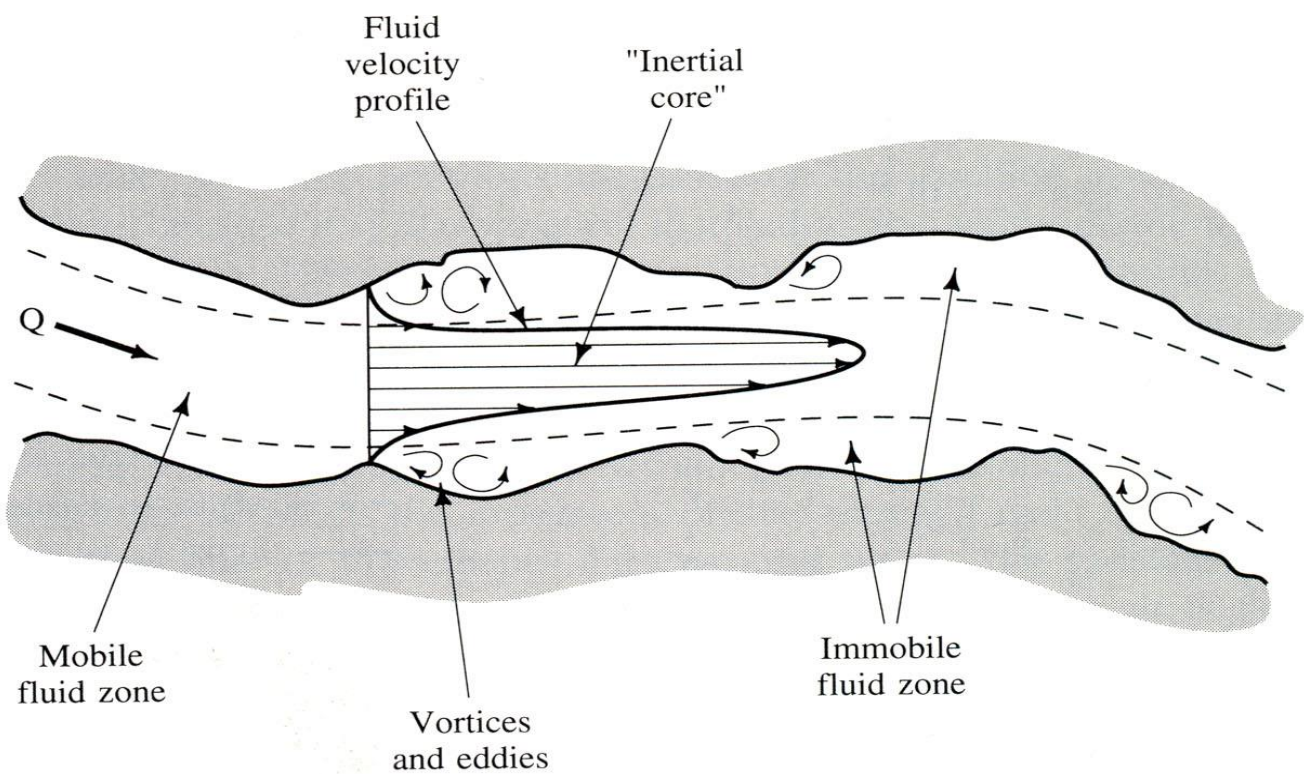
Dispersion in column experiment



(a)

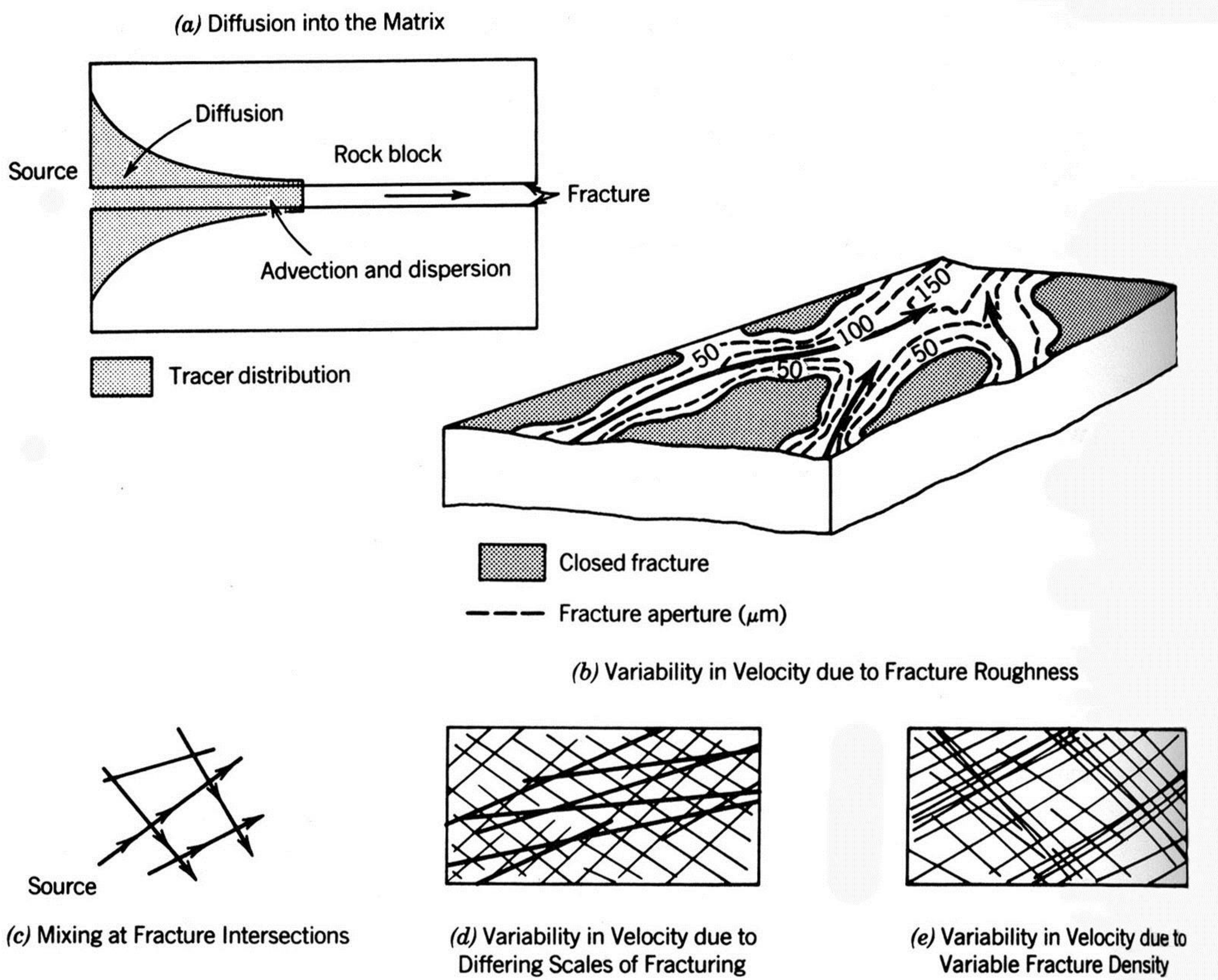
Experimental apparatus to illustrate dispersion in a column. The test begins with a continuous input of tracer $C/C_0 = 1$ at the inflow end. The relative concentration versus time function at the outflow characterizes dispersion in the column.

Immobile zones and dispersion



Zones of mobile and immobile water in a fracture. *Source:* K. G. Raven, K. S. Novakowski, and P. A. Lapcevic, *Water Resources Research* 24, no. 12 (1988): 2019–32. Published by the American Geophysical Union.

Scales of dispersion



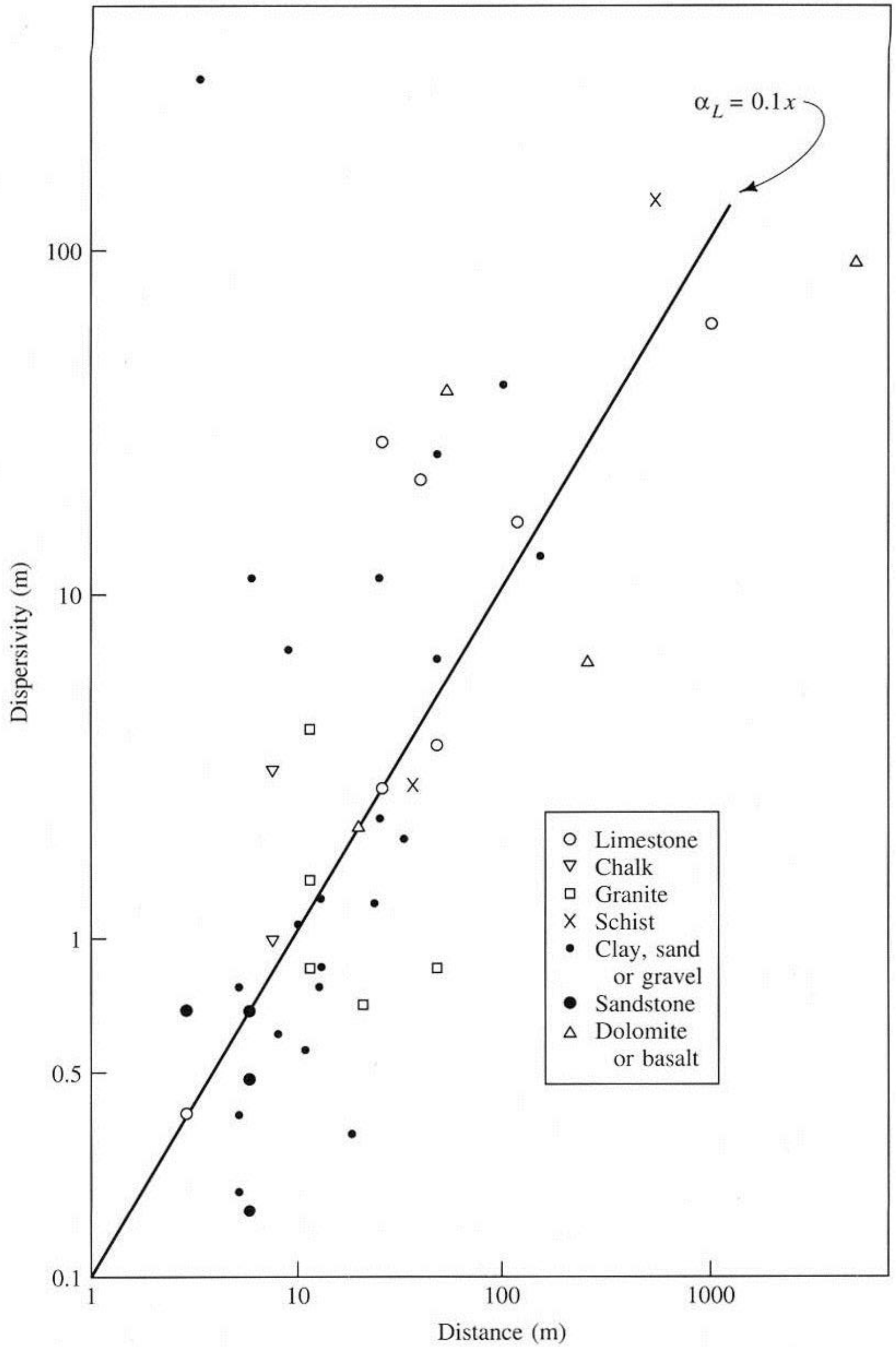
(a) Diffusion into the Matrix

(b) Variability in Velocity due to Fracture Roughness

(c) Mixing at Fracture Intersections

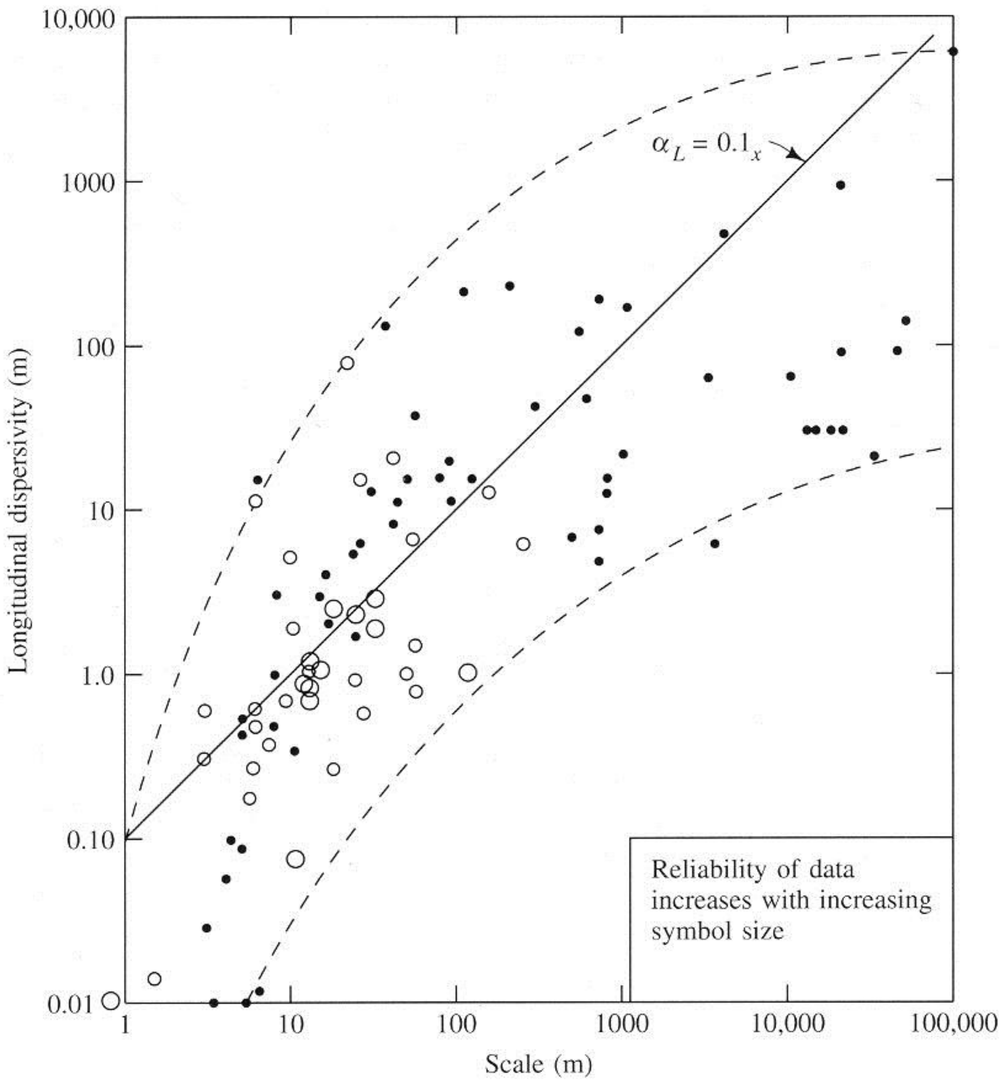
(d) Variability in Velocity due to Differing Scales of Fracturing

(e) Variability in Velocity due to Variable Fracture Density

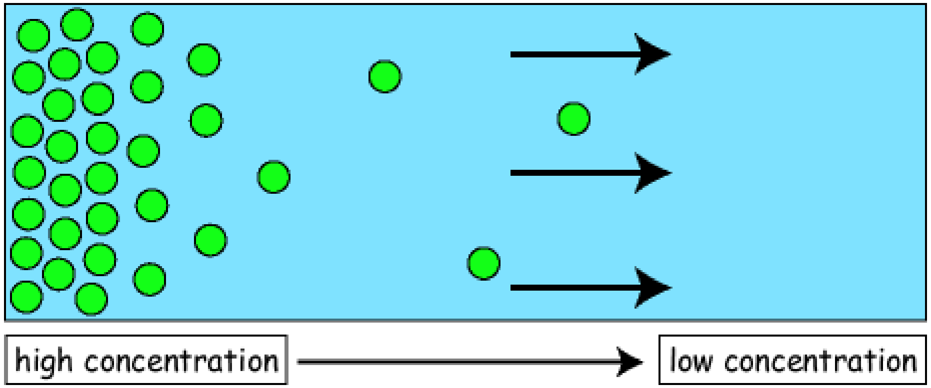


$\alpha_L = 0.1x$
Dispersivity (m)
100
10
1
0.5
0.1
1
10
100
1000
Distance (m)
Limestone
Chalk
Granite
Schist
Clay, sand or gravel
Sandstone
Dolomite or basalt

Scale effects



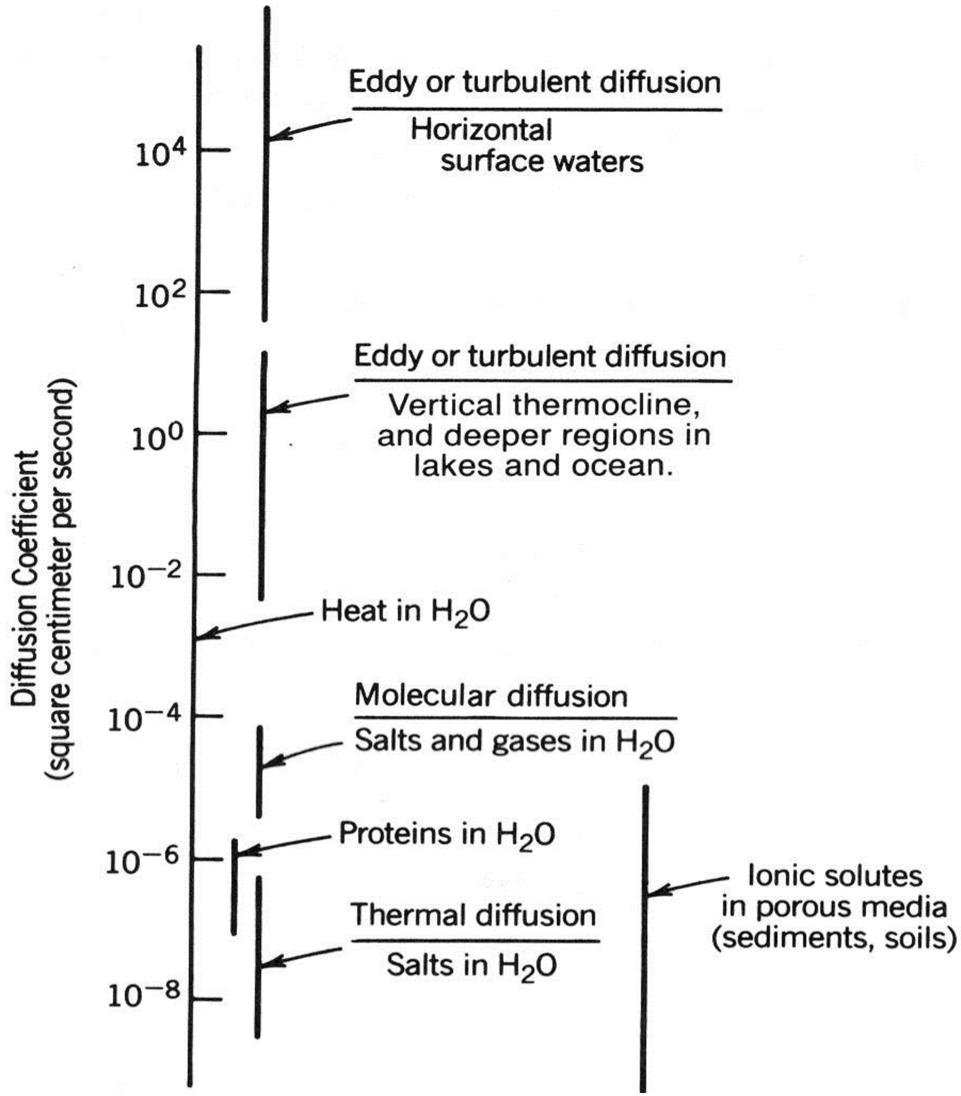
Diffusion



$$f^{*}_{dif} = -d_o \cdot i_c \qquad \text{mit} \qquad i_c = \frac{\partial c}{\partial x}$$

f^{*}_{dif}	Diffusiver Massenfluß im freien Wasservolumen [kg/(m²s)]
d_0	Diffusionskoeffizient im freien Wasservolumen [m²/s]
i_c	Konzentrationsgradient[kg/m⁴]
c	Konzentration des wassergelösten Inhaltsstoffes [kg/m³]
x	Abstand zwischen 2 Konzentrationsmeßpunkten [m]

Diffusion coefficient



Transport equation

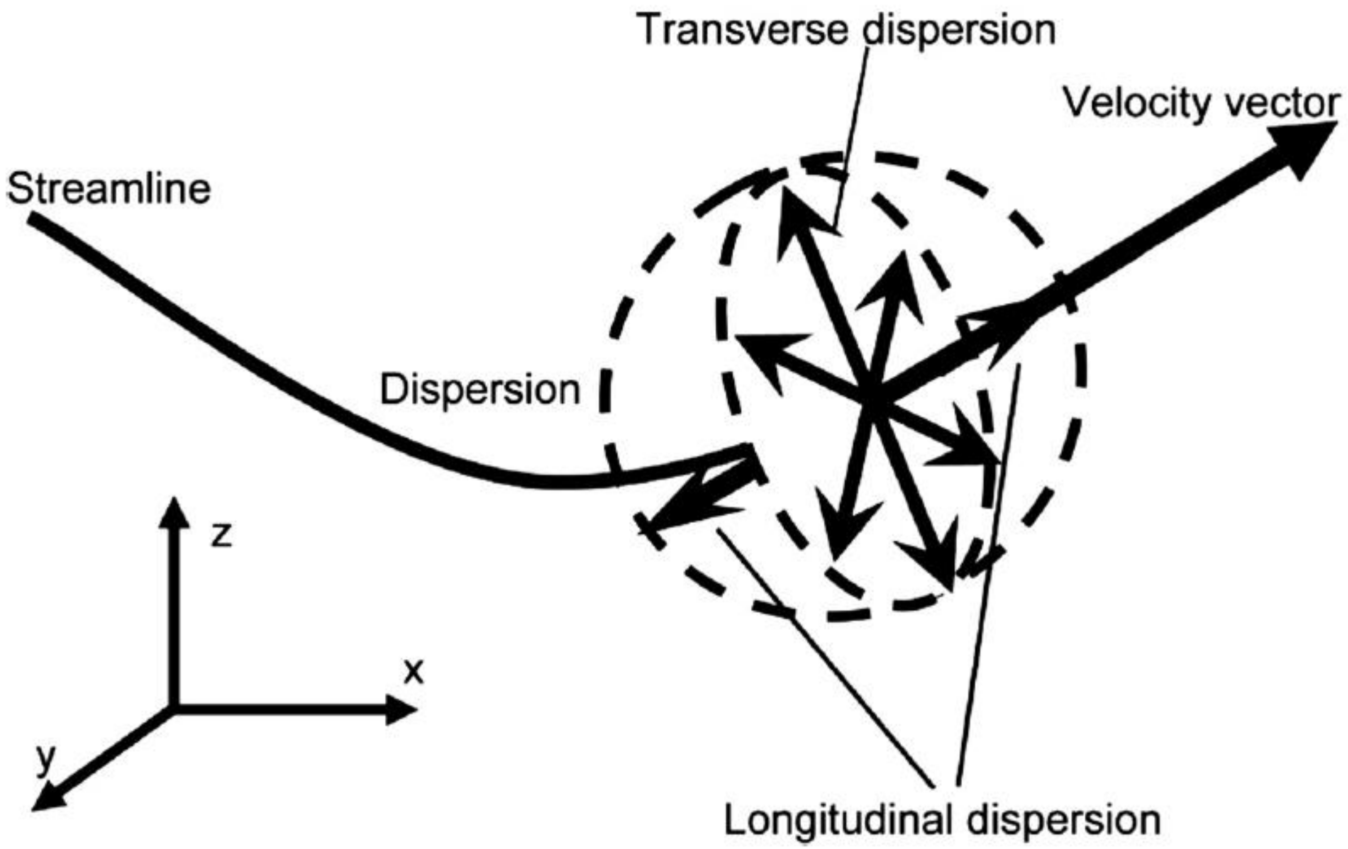
$$\frac{\partial}{\partial x}\left(D_{xx}\frac{\partial C}{\partial x}+D_{xy}\frac{\partial C}{\partial y}+D_{xz}\frac{\partial C}{\partial z}-v_x C\right)$$

$$+\frac{\partial}{\partial y}\left(D_{yx}\frac{\partial C}{\partial x}+D_{yy}\frac{\partial C}{\partial y}+D_{yz}\frac{\partial C}{\partial z}-v_y C\right)$$

$$+\frac{\partial}{\partial z}\left(D_{zx}\frac{\partial C}{\partial x}+D_{zy}\frac{\partial C}{\partial y}+D_{zz}\frac{\partial C}{\partial z}-v_z C\right)=\frac{\partial C}{\partial t}$$

- D__ dispersion coeffizient x,y,z direction
- C concentration t time
- v _flow velocity

Dispersion effects



Dispersivity and dispersion coefficient

$$D_L = \alpha_L \; v$$
$$D_T = \alpha_T \; v$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

- D dispersion coeffizient
  α dispersivity
- v flow velocity

Definition for Dispersivity and Tortuosity

$$D_{xx} = D_L\left(\frac{v_x^2}{v^2}\right) + D_T\left(\frac{v_y^2}{v^2}\right) + D_T\left(\frac{v_z^2}{v^2}\right) + \frac{D_m}{\tau}$$

$$D_{yy} = D_T\left(\frac{v_x^2}{v^2}\right) + D_L\left(\frac{v_y^2}{v^2}\right) + D_T\left(\frac{v_z^2}{v^2}\right) + \frac{D_m}{\tau}$$

$$D_{zz} = D_T\left(\frac{v_x^2}{v^2}\right) + D_T\left(\frac{v_y^2}{v^2}\right) + D_L\left(\frac{v_z^2}{v^2}\right) + \frac{D_m}{\tau}$$

$$D_{xy} = D_{yx} = (D_L - D_T)\left(\frac{v_x v_y}{v^2}\right)$$

$$D_{xz} = D_{zx} = (D_L - D_T)\left(\frac{v_x v_z}{v^2}\right)$$

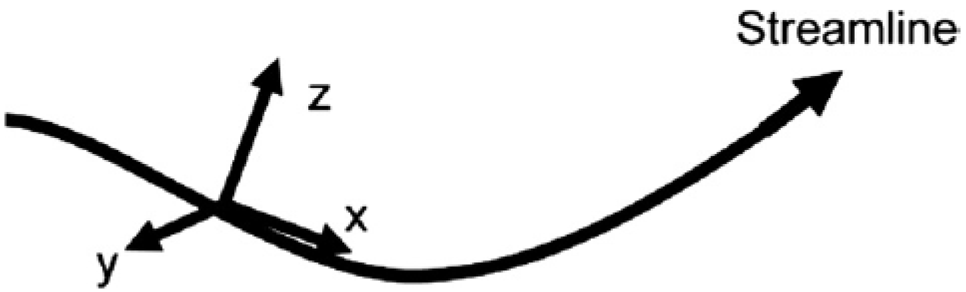
$$D_{zy} = D_{yz} = (D_L - D_T)\left(\frac{v_z v_y}{v^2}\right)$$

- T Tortuosity
- D_ Dispersion coefficienten
- L, T for lateral and transversal

3D

3D equation

$$D_{xx}\frac{\partial^2 C}{\partial x^2} + D_{yy}\frac{\partial^2 C}{\partial y^2} + D_{zz}\frac{\partial^2 C}{\partial z^2} - v\frac{\partial C}{\partial x} = \frac{\partial C}{\partial t}$$



- 3D Transport
- Translation and dispersion

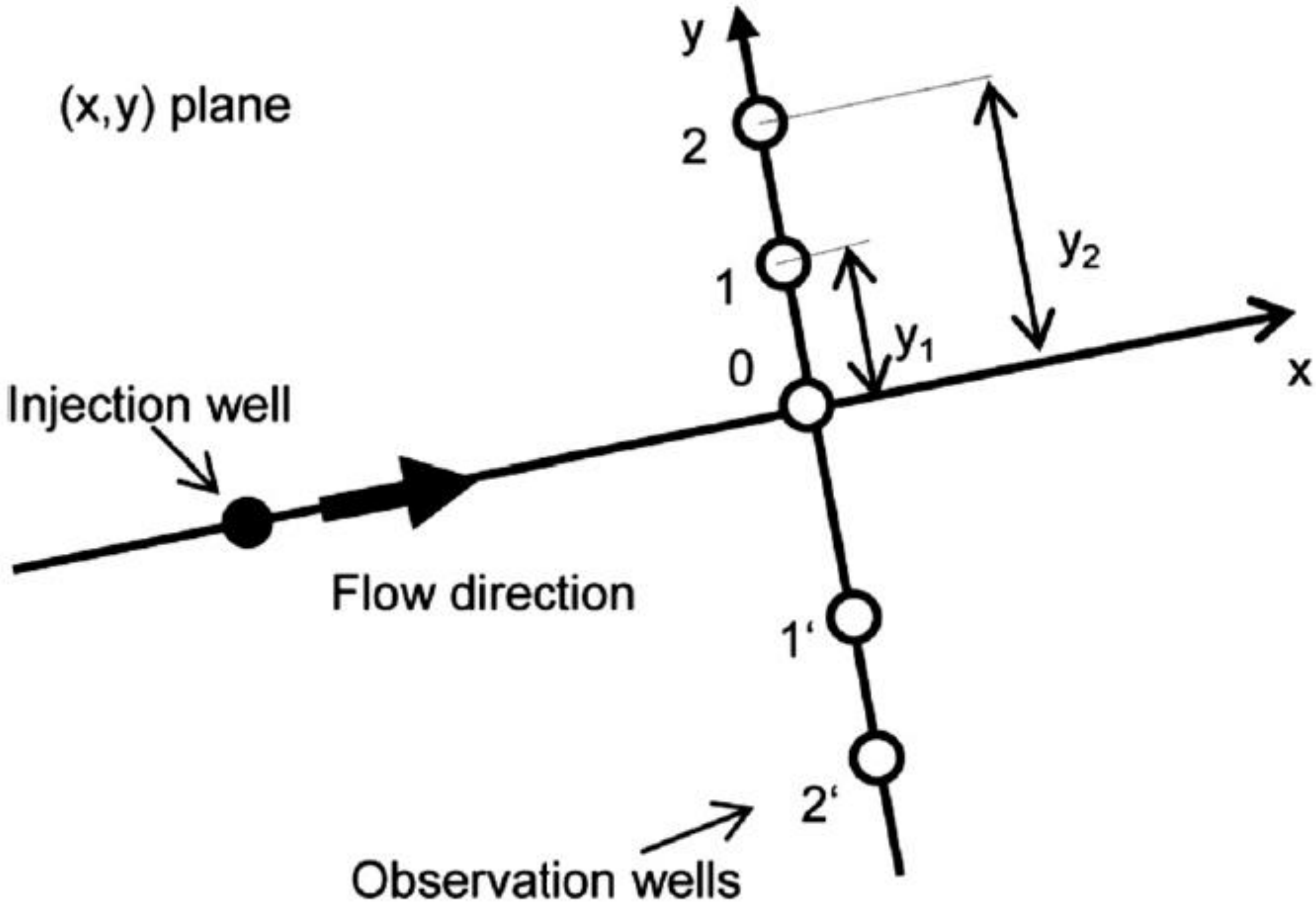
3D

$$D_{xx} = D_L + \frac{D_m}{\tau}$$
$$D_{yy} = D_T + \frac{D_m}{\tau}$$
$$D_{zz} = D_T + \frac{D_m}{\tau}$$

- $v < 0.1$ m/Tag :
  - D_L = Dxx
  - D_T = Dyy = Dzz

2D

Boundary conditions



2D

if

$$\frac{\partial C}{\partial z} = 0$$

then

$$D_L \frac{\partial^2 C}{\partial x^2} + D_T \frac{\partial^2 C}{\partial y^2} - v \frac{\partial C}{\partial x} = \frac{\partial C}{\partial t}$$

Boundary conditions

$$C(x=0,\, y=0,\, t) = \frac{M}{nH}\,\delta(t)\,\delta\,(x)\,\delta(y)$$

$$C(x,\, y,\, t=0) = 0$$

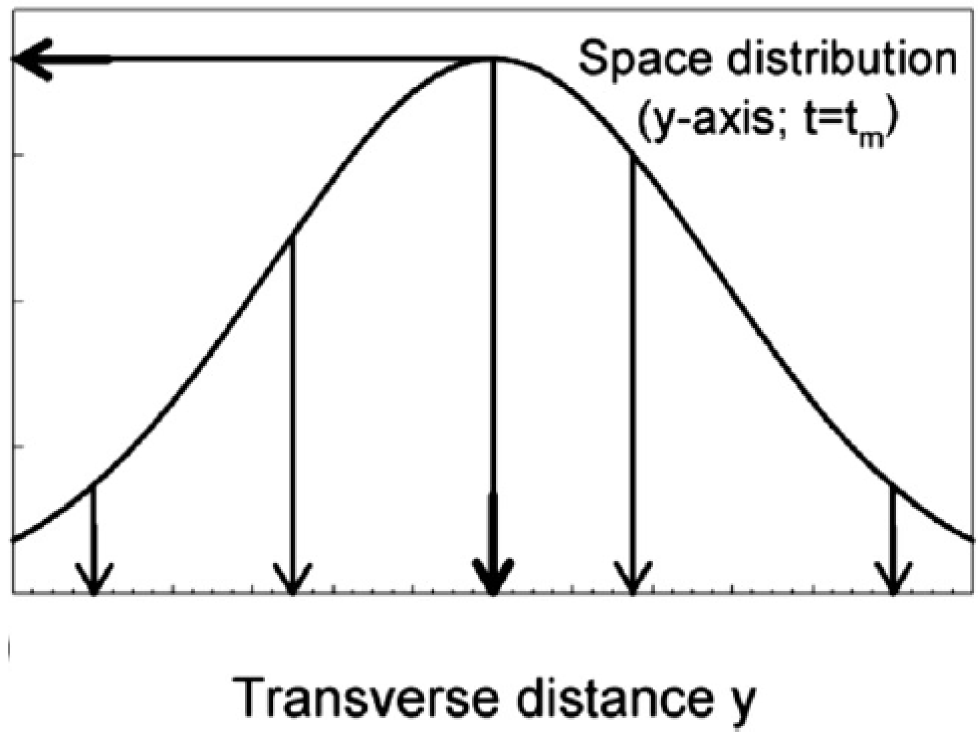
$$\lim_{(x,\, y) \to \infty} C(x,\, y,\, t) = 0$$

- *M mass (g)*
- *‚n' porosity*
- *H is thickness*

Analytical solution 2D

$$C(x, y, t) = \frac{M}{nH} \frac{x}{4\pi v t^2 \sqrt{D_L D_T}} \exp\left[-\frac{(x - vt)^2}{4D_L t} - \frac{y^2}{4D_T t}\right]$$

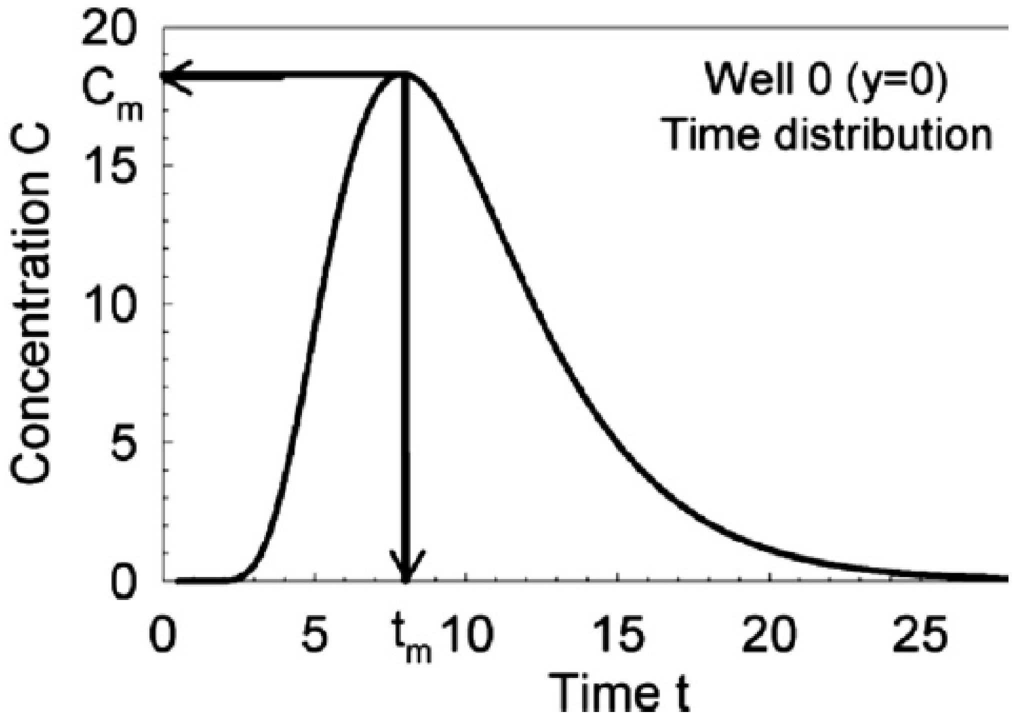
- *M is mass (g)*
- *n is porosity*
- *H is thickness*
- *D_L is longitudinal D_T is transversal Dispersion*



Time equation 2D

$$C(t) = C_m \left(\frac{t_m}{t}\right)^2 \exp\left[-\frac{(x - vt)^2}{4D_L\, t} + \frac{(x - vt_m)^2}{4D_L\, t_m}\right]$$

- *tm is residence time*



Transverse dispersion

$$C(y) = C_m \exp\left[-\frac{y^2}{4D_T t_m}\right]$$

- Y is transverse distance
- D_T is transversal dispersion coeffizient
- t_m is residence time

Finally …

1D

Momentaneous injection

- Dirac-Stoß

1D

if

$$\frac{\partial C}{\partial y} = \frac{\partial C}{\partial z} = 0$$

then:

$$D_L \frac{\partial^2 C}{\partial x^2} - v\frac{\partial C}{\partial x} = \frac{\partial C}{\partial t}$$

- 1D no change in y and z
- Rivers and columns

Boundary conditions 1D

$$C(x=0,\, t) = \frac{M}{Q}\delta(t)$$

$$C(x,\, t=0) = 0$$

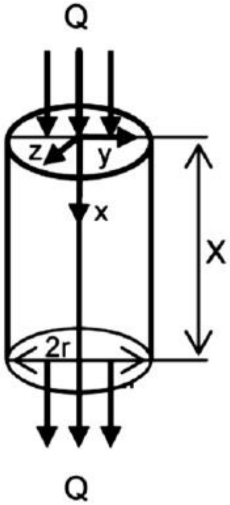
$$\lim_{x \to \infty} C(x,\, t) = 0$$

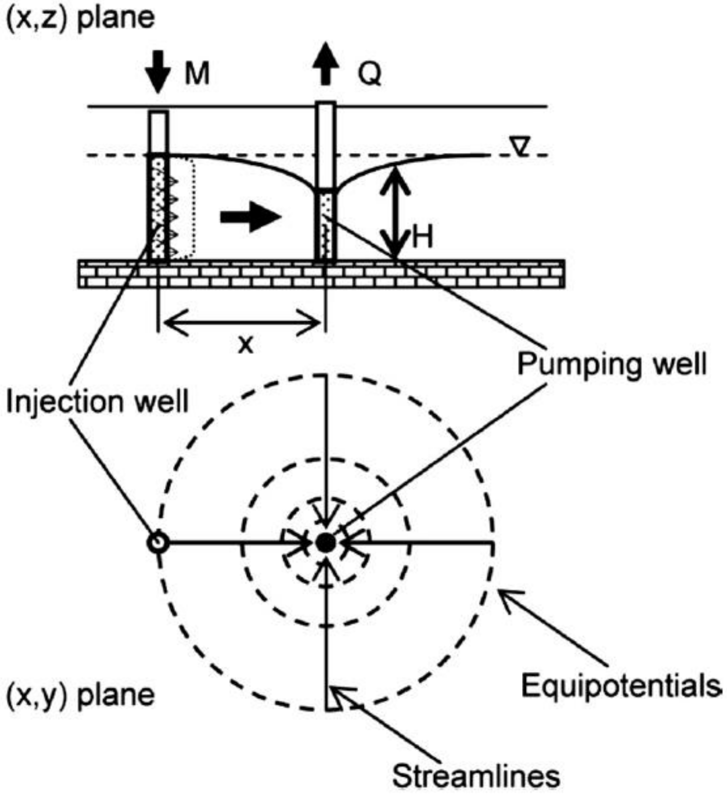
Solution 1D

$$C(x,t) = \frac{M}{Q}\frac{x}{\sqrt{4\pi D_L t^3}} \exp\left[-\frac{(x-vt)^2}{4D_L t}\right]$$

- Main equation 1D
- C depends on M and 1/Q and $D_L^{-1/2}$ and $1/t^{3/2}$ and decreases exponentially with x^2

Radial 1D





- Applicable to wells

Residence time

$$t_0 = \frac{x}{v}$$

$$t_0 = \frac{V}{Q}$$

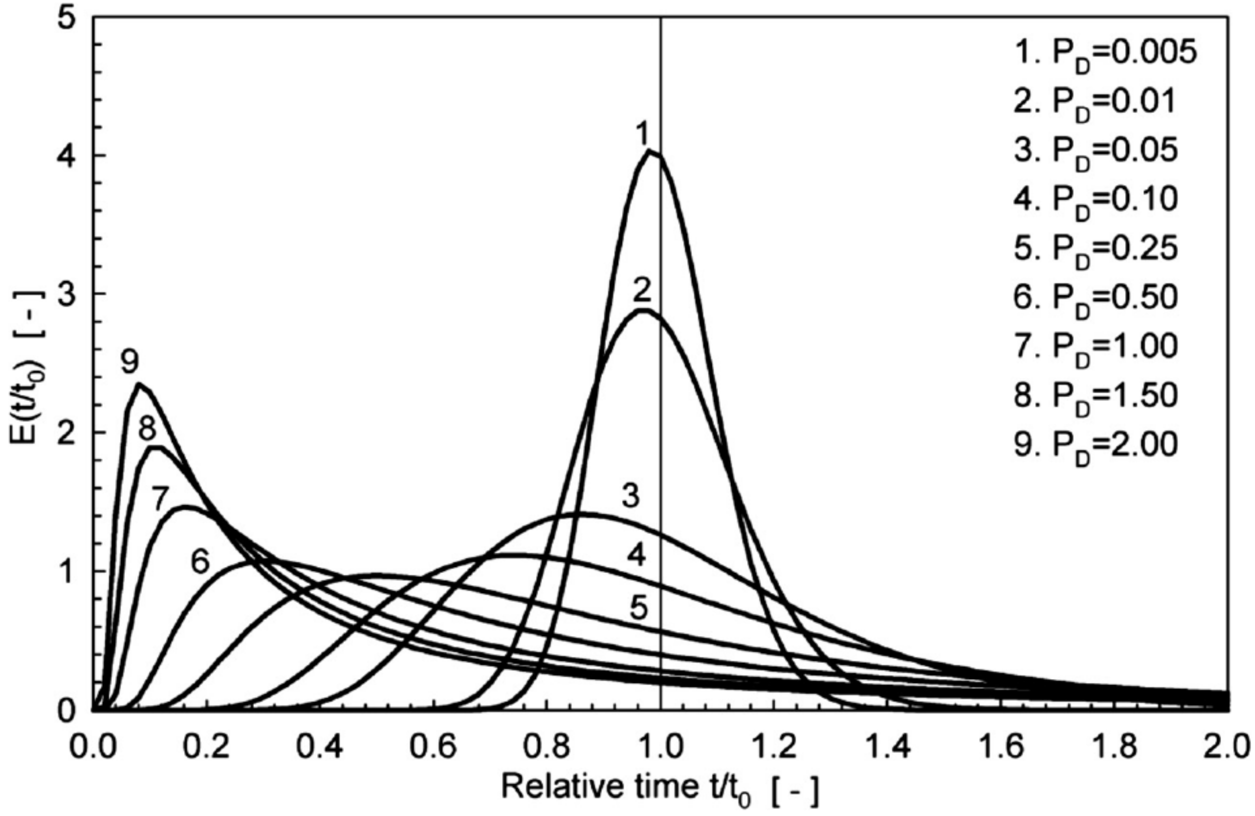
- In proportion to Volume and
- In inverse proportion to discharge

Dispersion parameter (Definition)

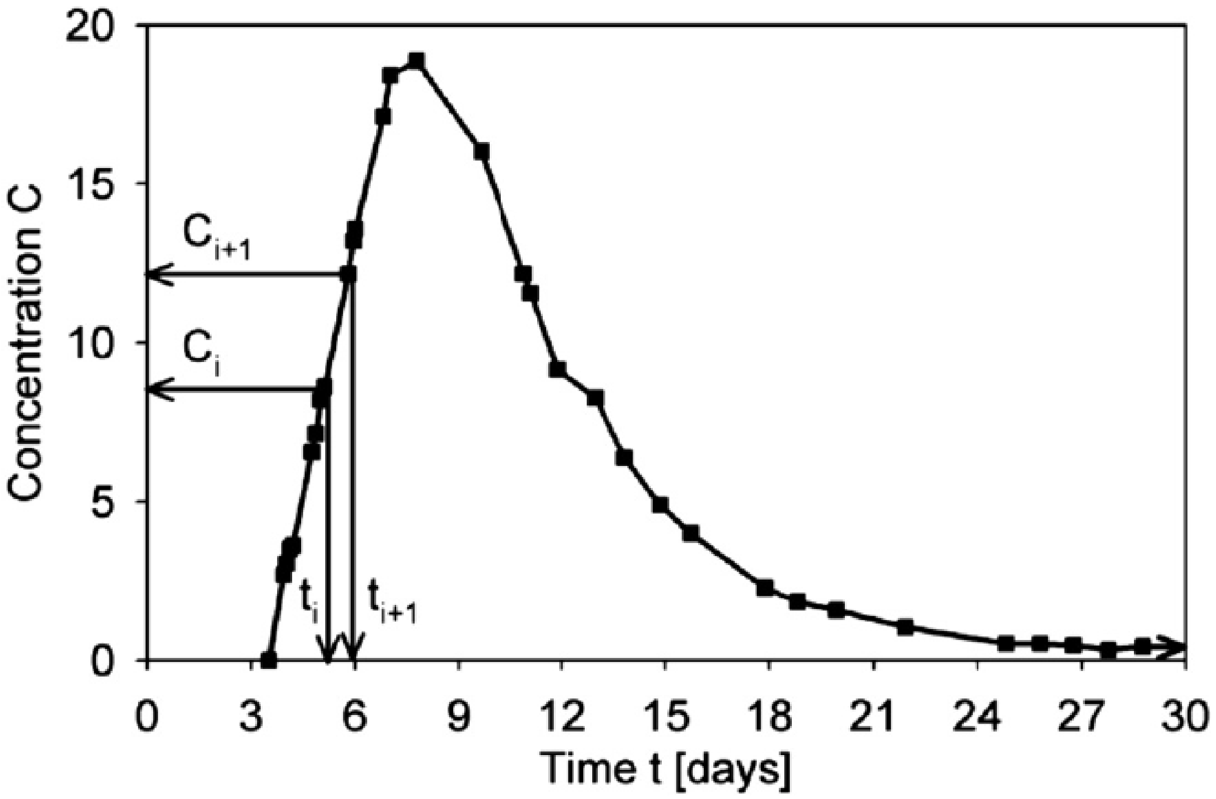
$$P_D = \frac{D_L}{v x} = \frac{\alpha_L}{x}$$

- Dispersion coeffizient per velocity
- Dispersivity per length x
- Dispersion parameter corresponds to α/x

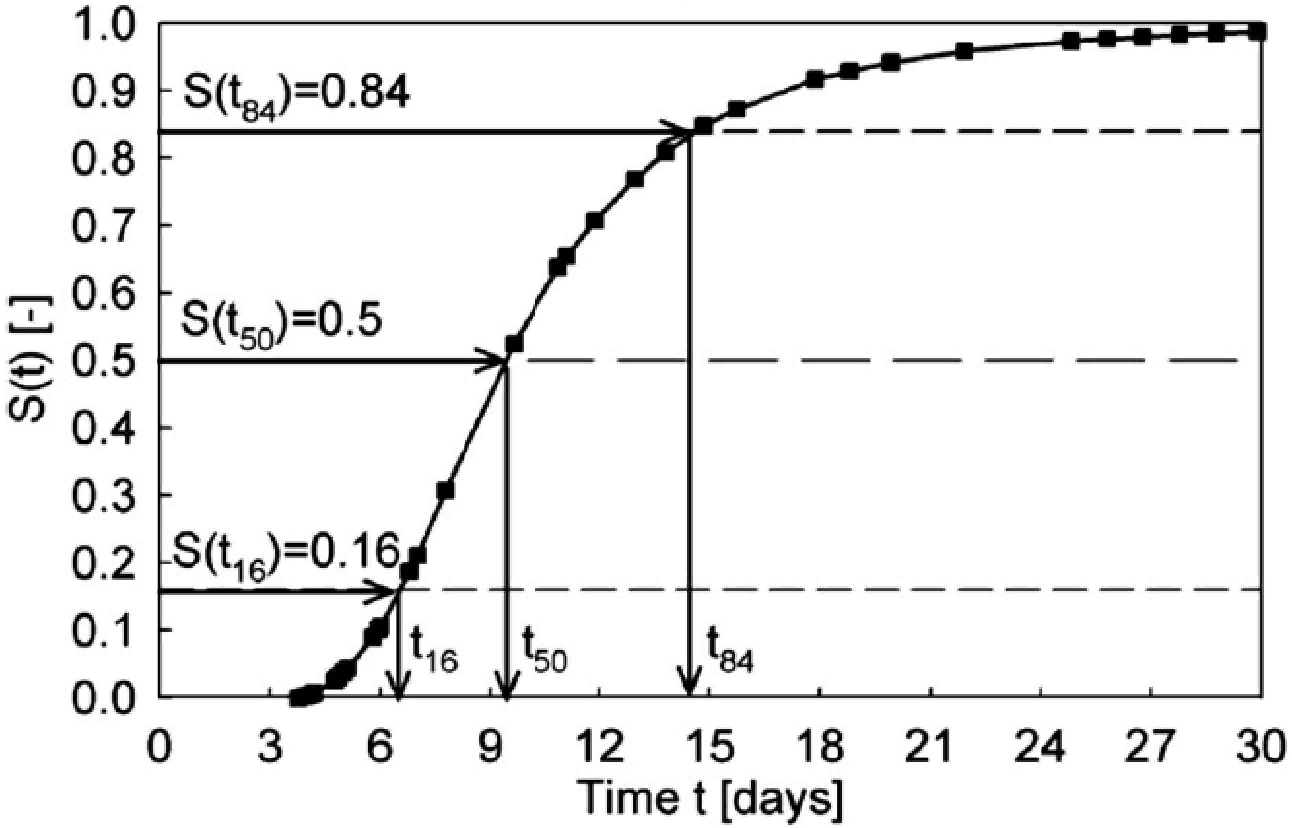
Normalized curves



Measurements

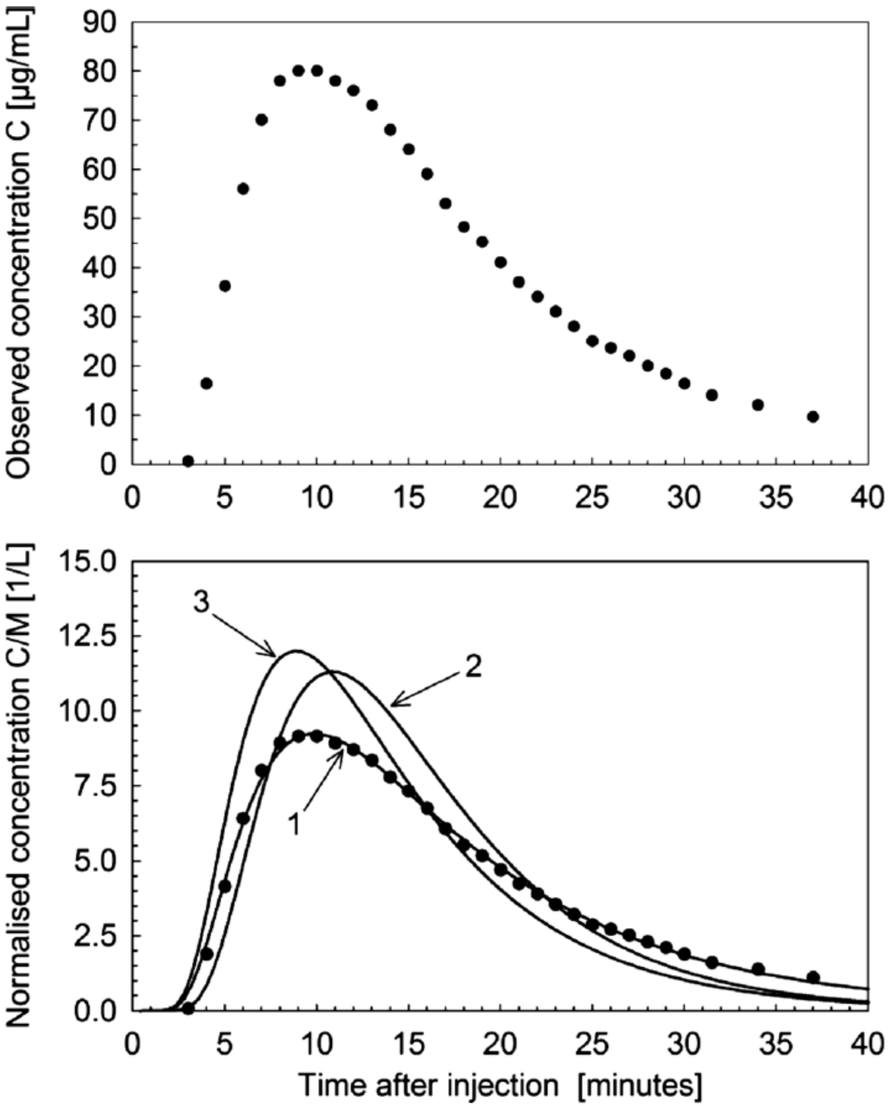


Mass curve



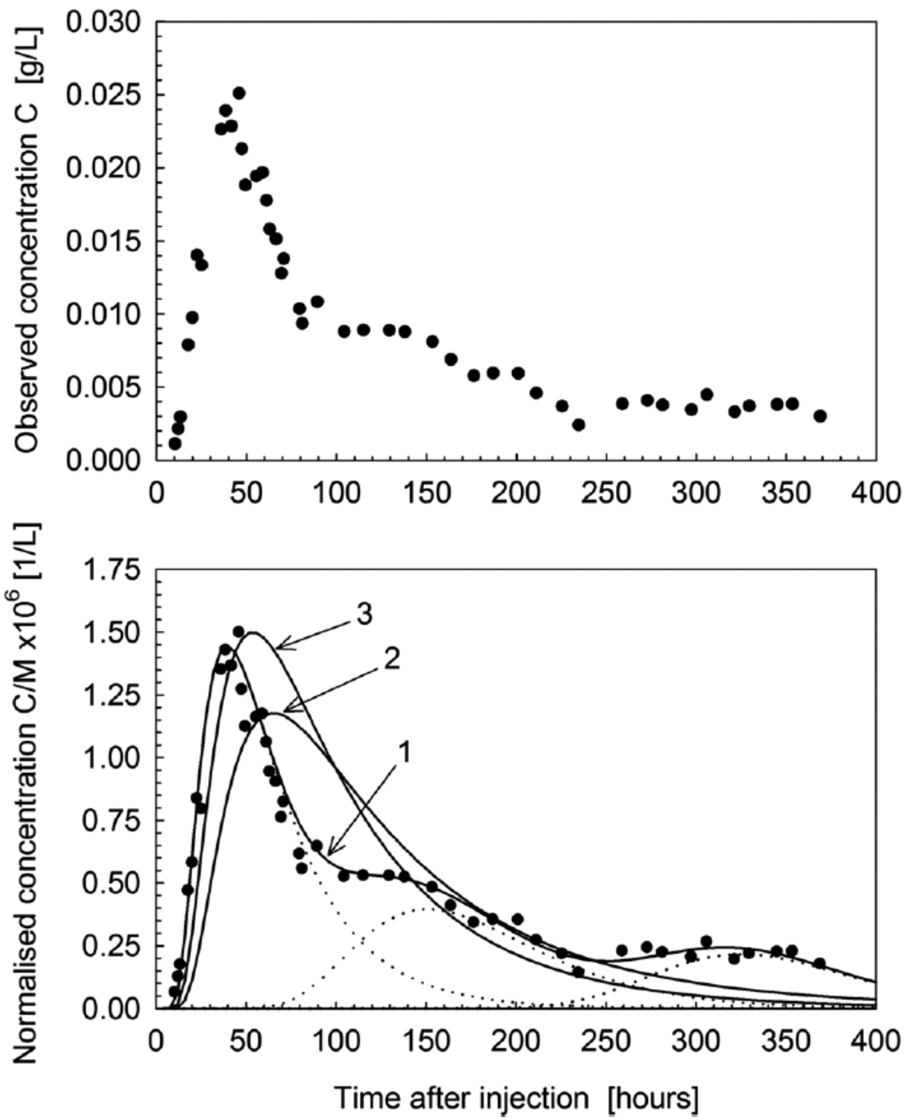
S(t_84)=0.84
S(t_50)=0.5
S(t_16)=0.16
t_16
t_50
t_84
S(t) [-]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
3
6
9
12
15
18
21
24
27
30
Time t [days]

Fitting

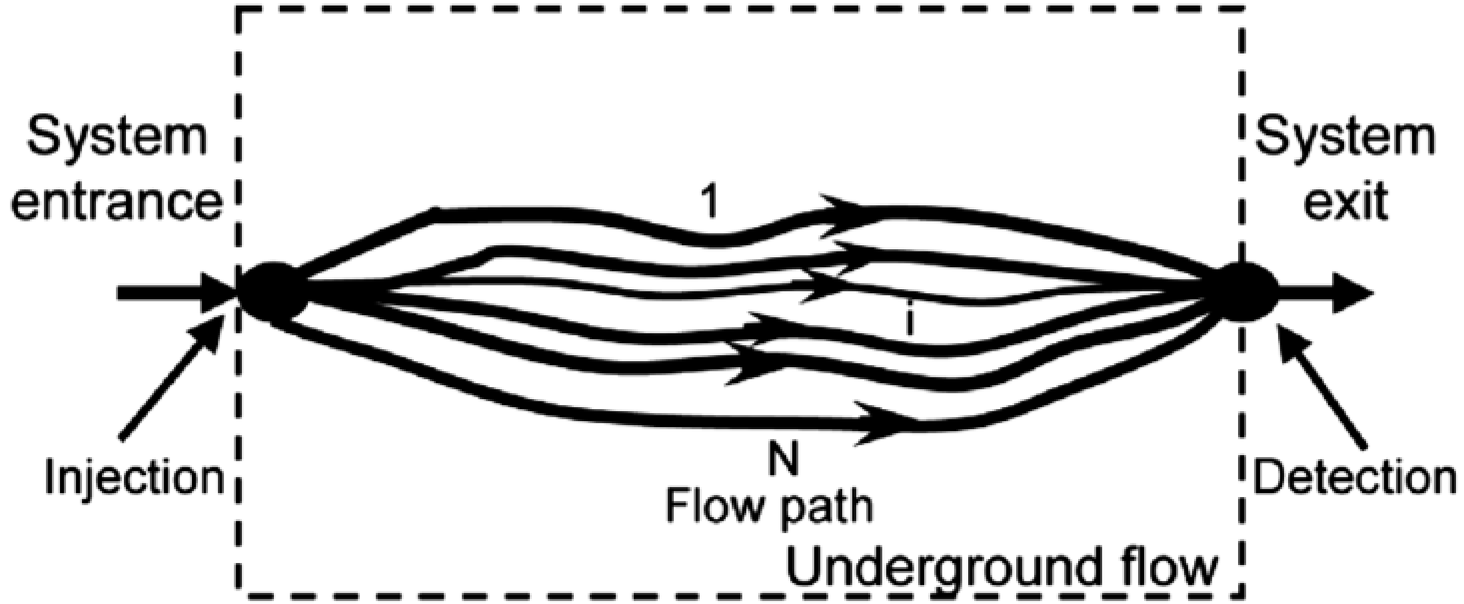


Observed concentration C [µg/mL]
90
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
Normalised concentration C/M [1/L]
15.0
12.5
10.0
7.5
5.0
2.5
0.0
3
2
1
Time after injection [minutes]

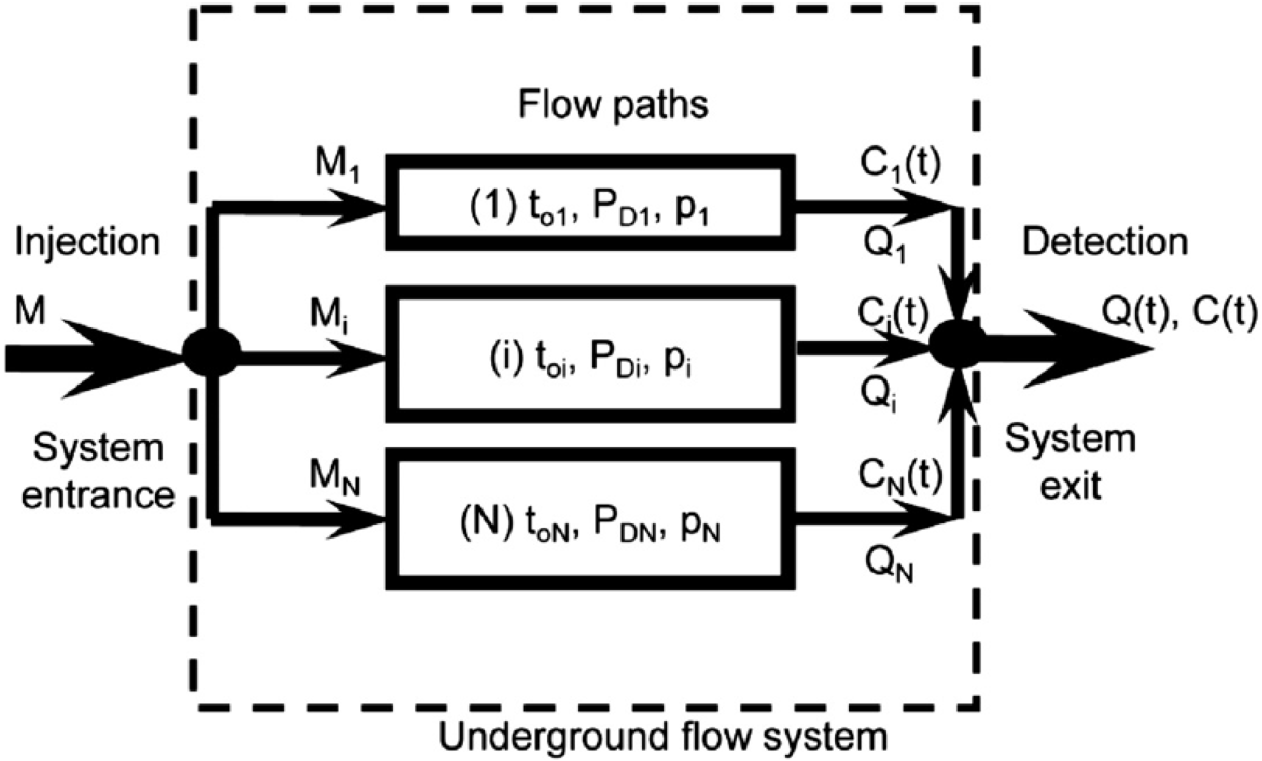
Multifinalität



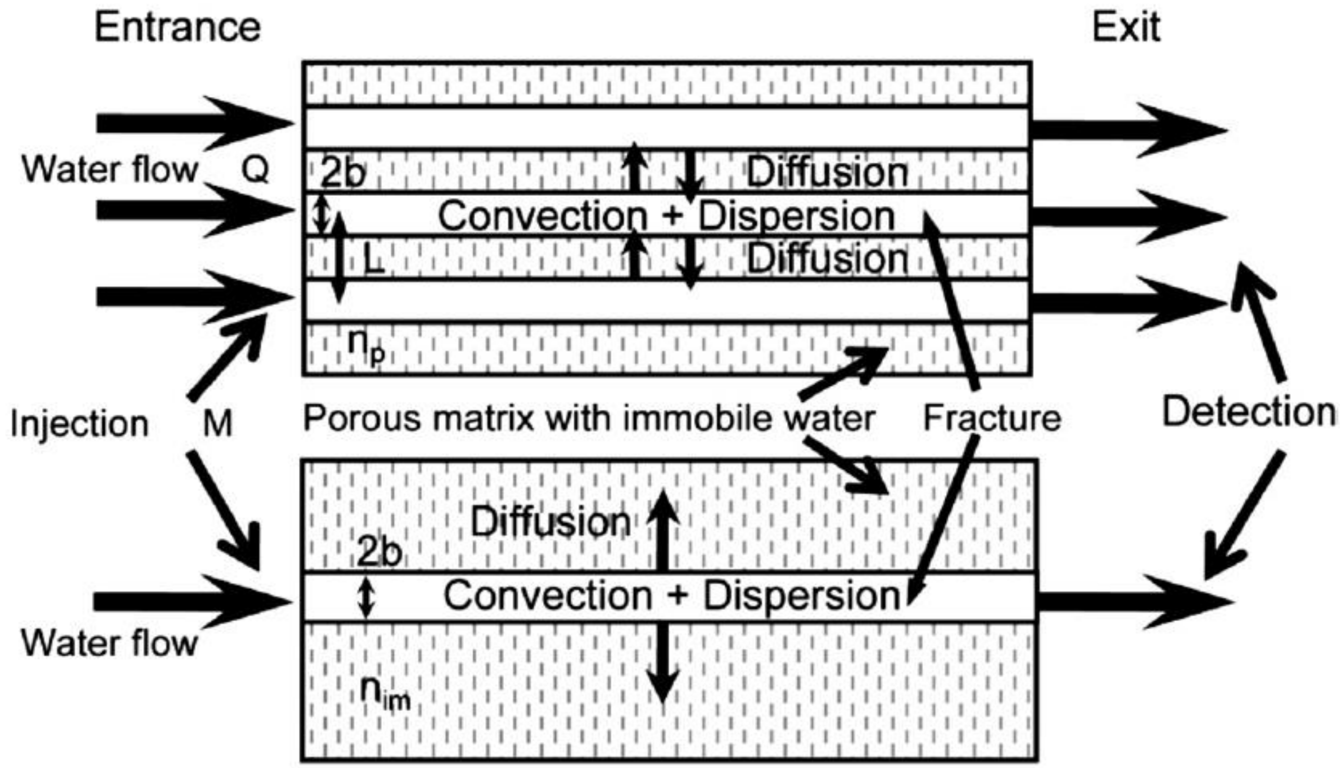
Conceptual model



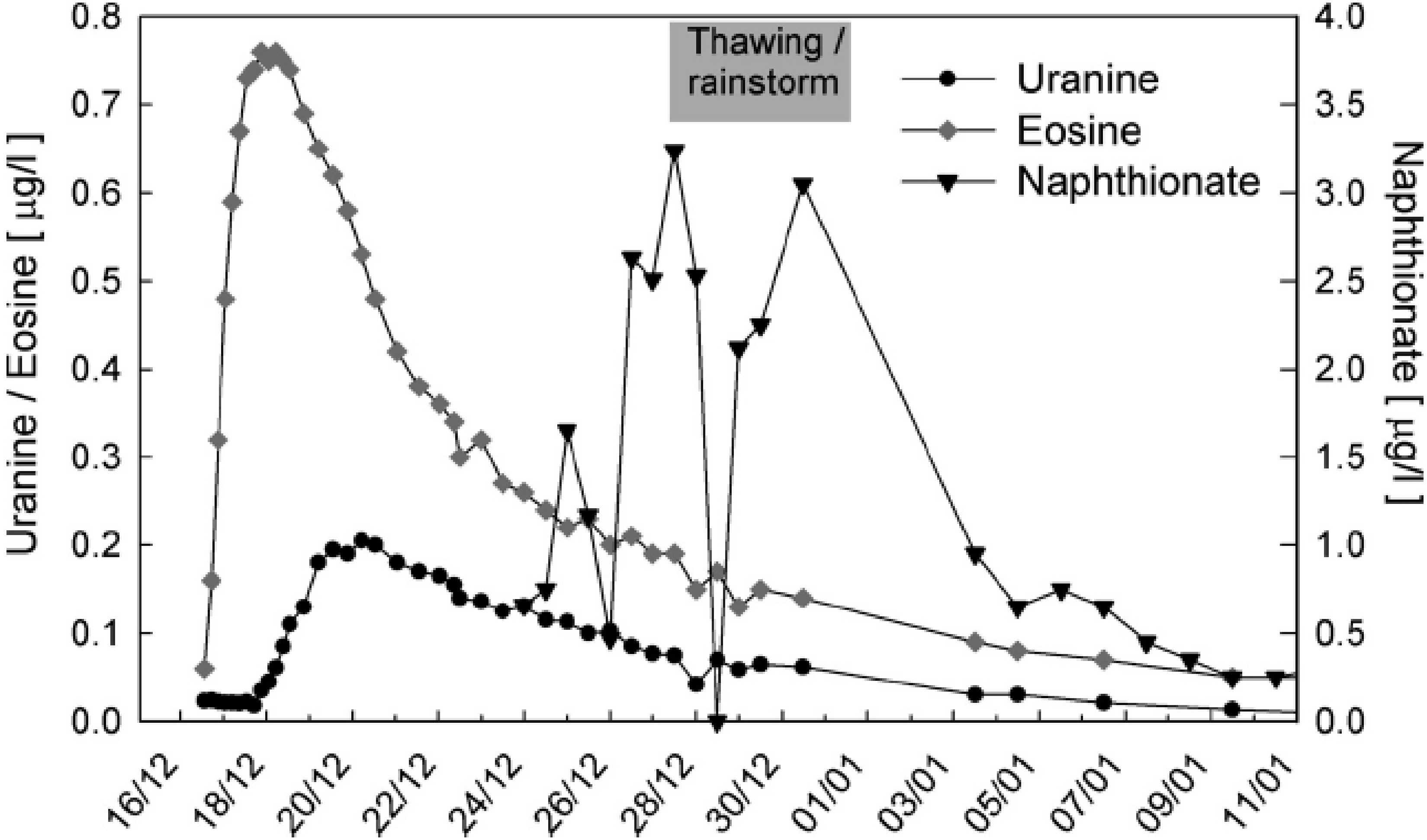
Parallel ADV Models



Dispersion and Diffusion



Example



Thawing / rainstorm
Uranine
Eosine
Naphthionate
Uranine / Eosine [µg/l]
Naphthionate [µg/l]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
16/12
18/12
20/12
22/12
24/12
26/12
28/12
30/12
01/01
03/01
05/01
07/01
09/01
11/01

Rhein

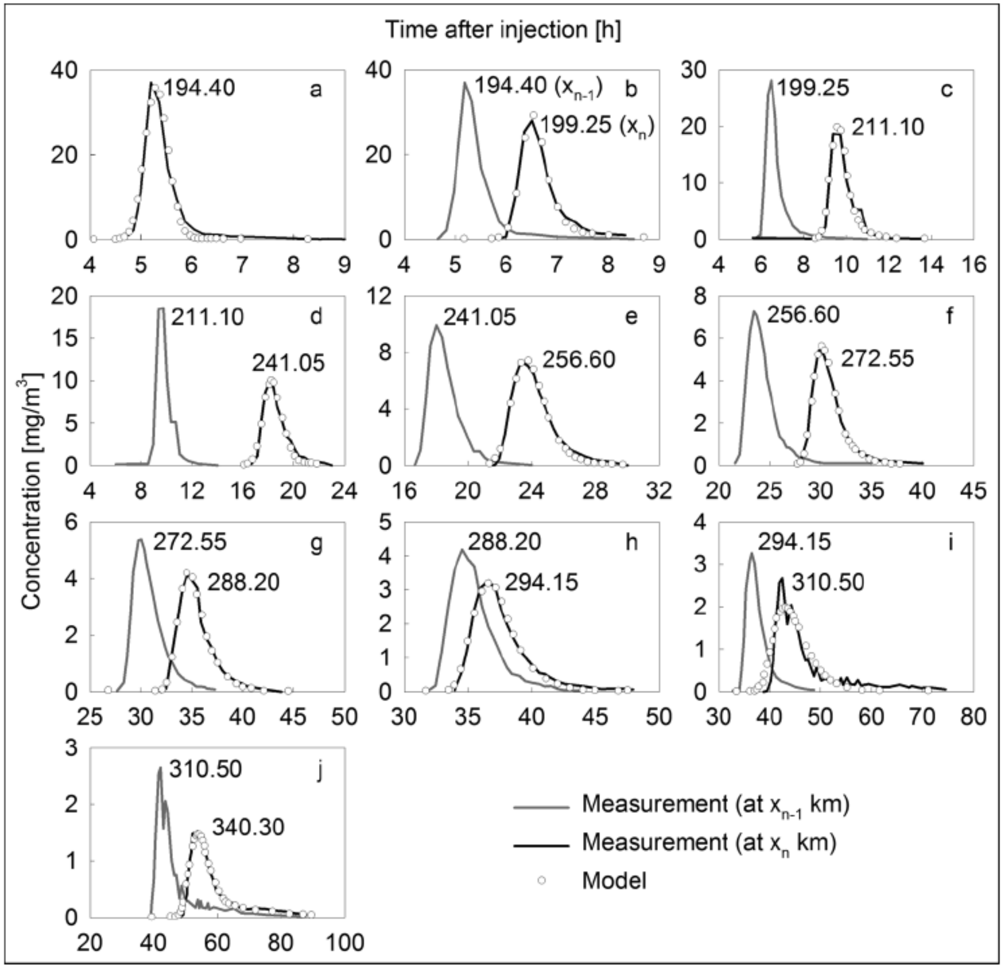


Figure 7.24 Experiment 04/89. Measured tracer breakthroughs at the following river sections (x_n): Ottmarsheim (km 194.4), Neuenburg (km 199.25), Fessenheim (km 211.1), Marckolsheim (km 241), Rheinau (km 256.6), Gerstheim (km 272.5), Strasbourg (km 288.2), Kehl (km 294.1), Gambsheim (km 310.5), Plittersdorf (km 350.3).

Fitting of models

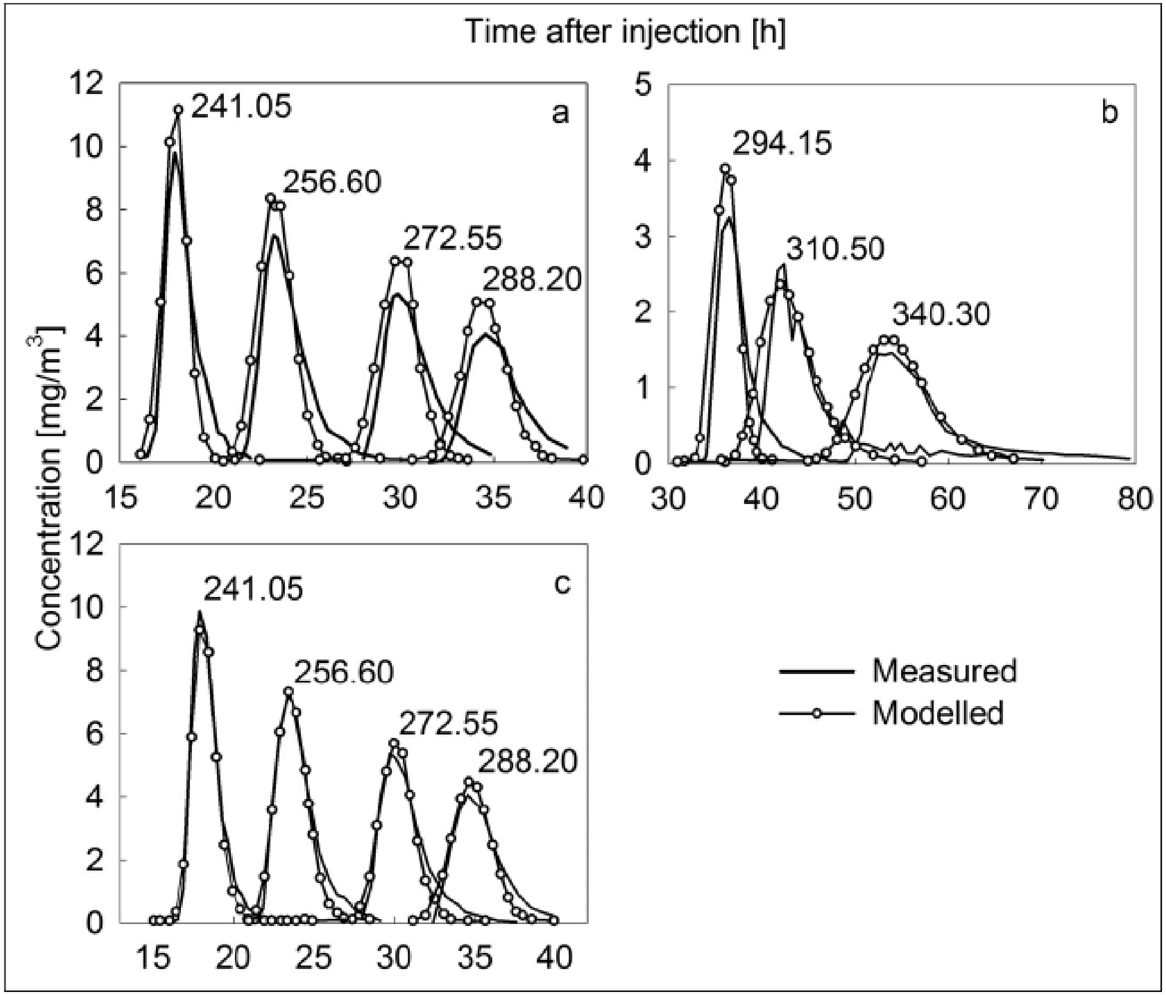
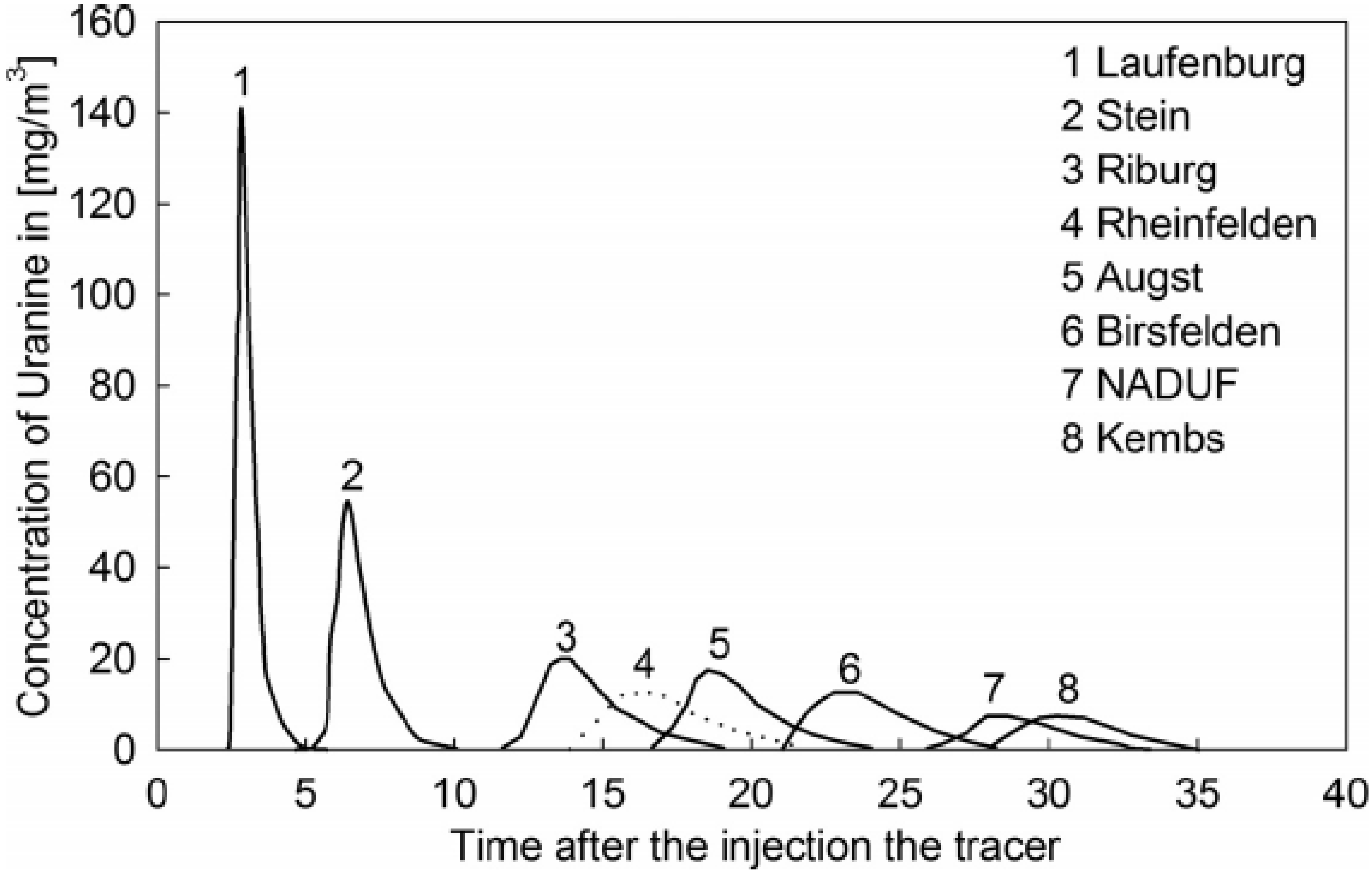
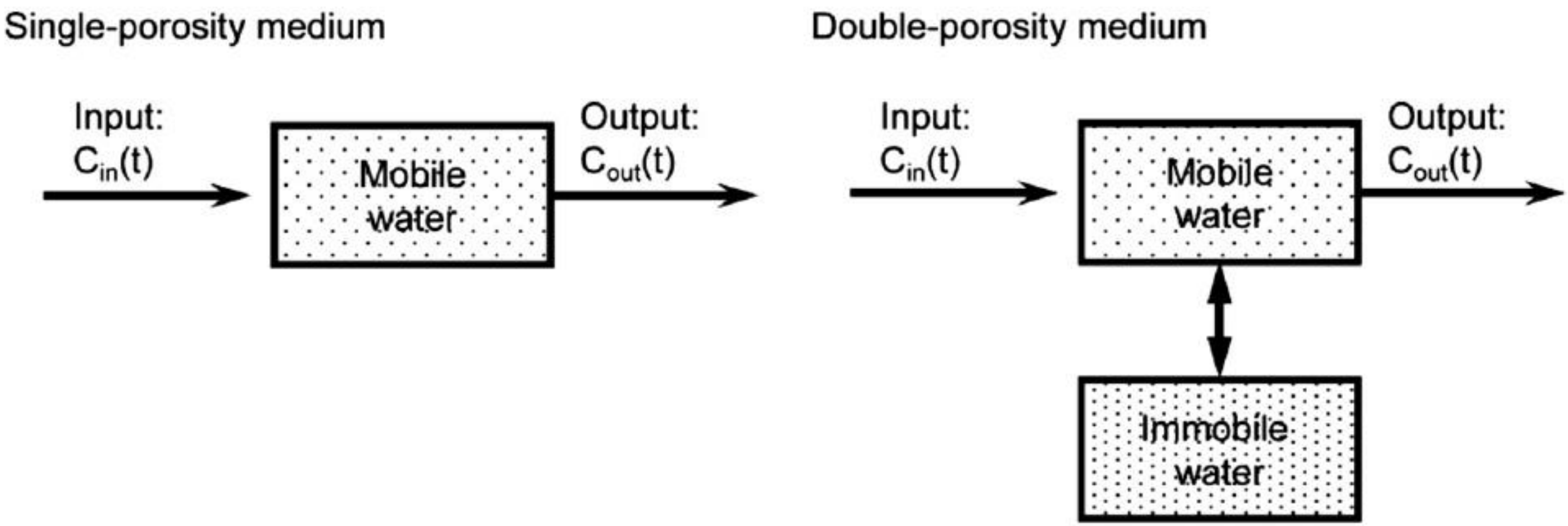


Figure 7.25 Validation of the Upper Rhine river sections: a) postmodelling, Rhine km 241.05–288; b) postmodelling, Rhine km 294.15–340.3; c) forecast improvement, Rhine km 241.05–288.2.

Warning model

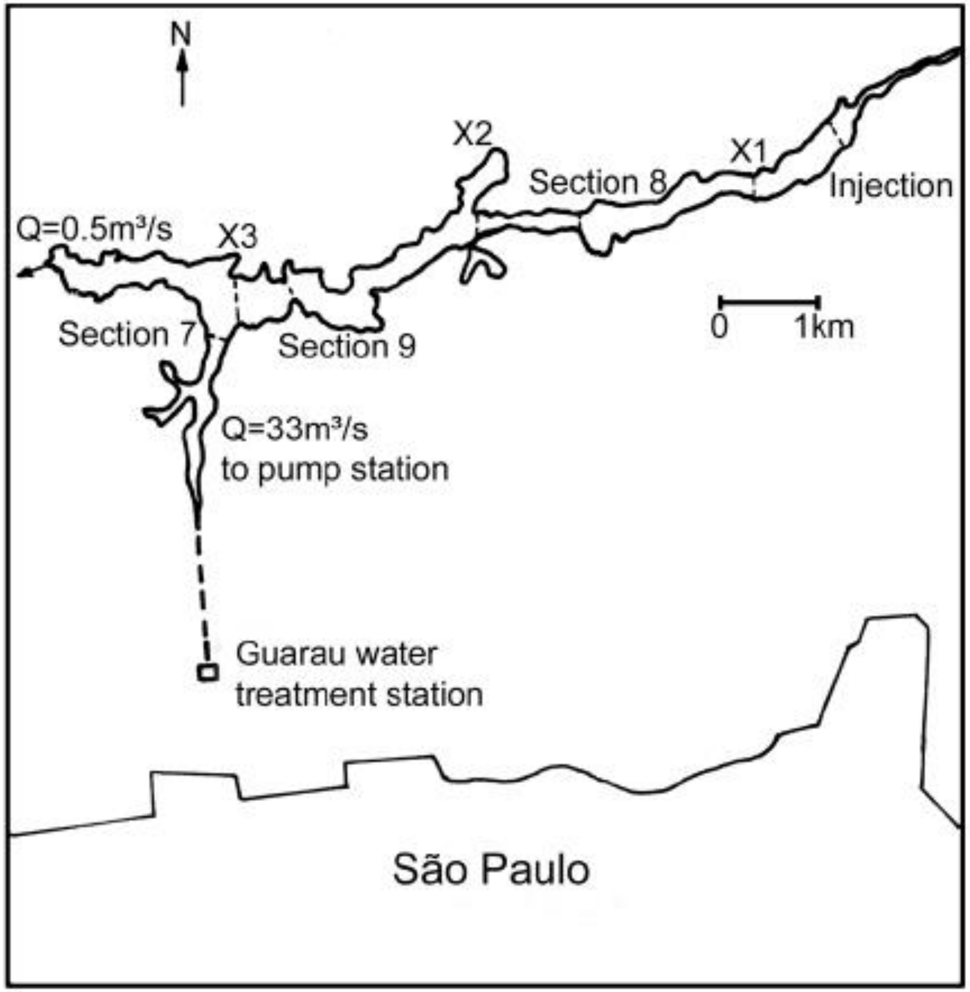


Combining advection dispersion and diffusion into dead zones

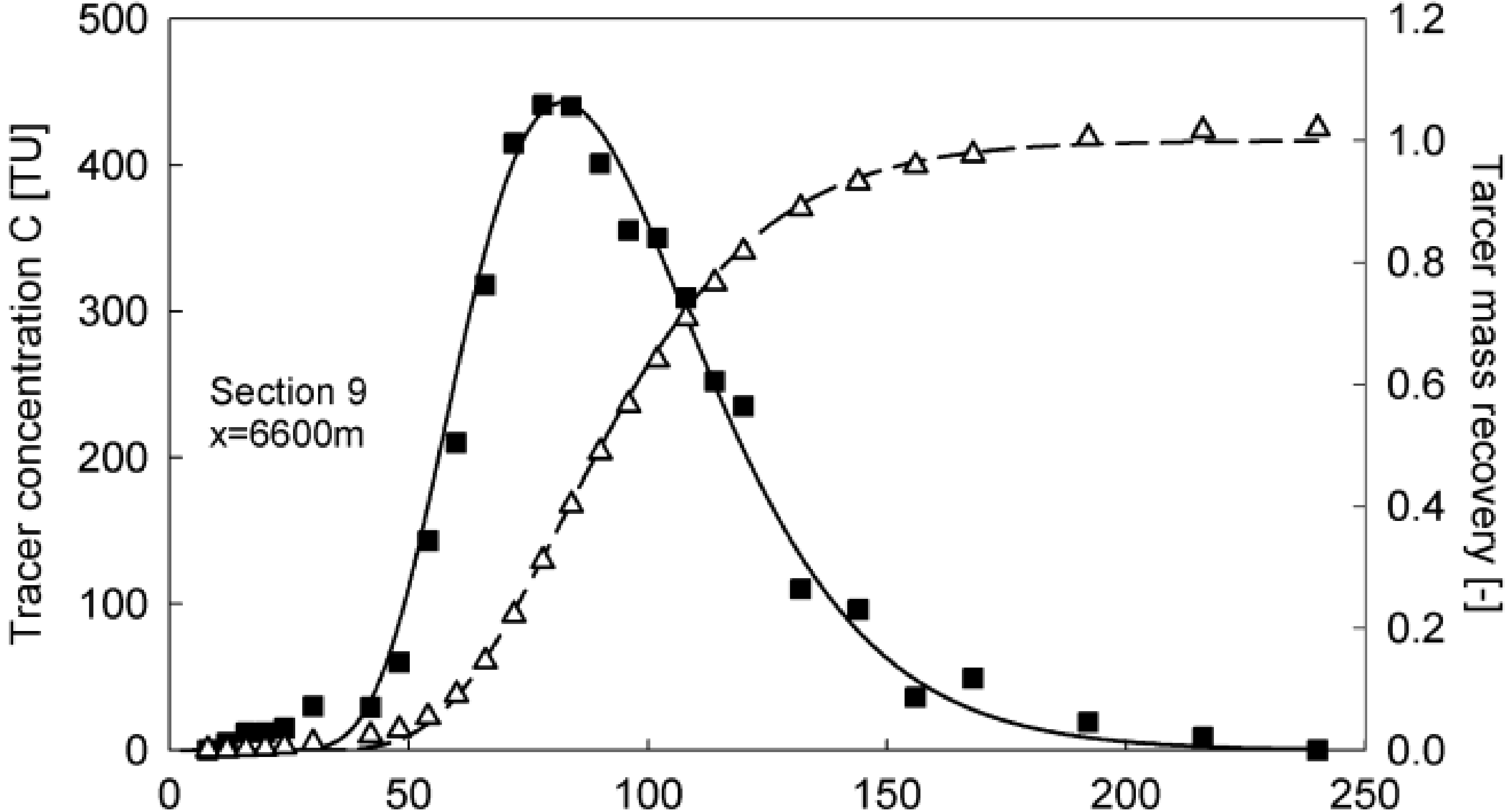


Brasil


50
40
BRAZIL
São Paulo
20
30


N
X2
X1
Section 8
Injection
Q=0.5m³/s
X3
0
1km
Section 7
Section 9
Q=33m³/s
to pump station
Guarau water
treatment station
São Paulo

Determining the residence time of water in a surface reservoir in Brasil near Sao Paulo



Summary

- ADV has analytical solutions for 1D, 2D
- Can be used to predict breakthrough
- Can be used to calculate mass M
- Fundamental equations for predicting mass transport