Boussinesq equation

Based on the Dupuit approximation some modifications can be made for a sloping aquifer, as first proposed by Boussinesq. Boussinesq proposed a modified version of Darcys law:

\$\$q_{GW}=-K_h*h*{\delta h}/{\delta x}*cos(\beta)-K_h*h*sin(\beta)\$\$

where q_{GW} is the groundwater flow to the stream, K_h is the hydraulic conductivity of the aquifer along the slope, h is the groundwater level above the aquifer base, \pm is the slope of the aquifer. For the meaning of variables see the sketch below:



When combined with the continuity equation, the so-called 'Boussinesq-Equation' is obtained:

Brutsaerts simplification

Brutsaert (1994) linearized the equation:

 $q_{GW}=-\lambda h^*h_0^{frac} delta h_{\lambda elta x}^{cos(\lambda eta)-K_h^*h^sin(\lambda eta)$

where $\lambda = 0.5$ and adjust for the use of h_0 instead of h_0 .

The Boussinesq equation can be simplified to:

The shape parameter

There is a shape parameter that describes the ratio between advection and diffusion terms:

 $J = \frac{K_h \sin \theta X}{S_y}}{\frac{S_y}} = \frac{S_y}{\frac{S_y}} = \frac{S_y}{S_y}{S_y}{S_y}{S_y}{S_y}{S_y}}S$

A large \$J\$ means steep slopw and shallow aquifer and a predomincance of advection, a small \$J\$ means shallow slope and thick aquifer and a dominance of diffusion, for a slope of \$\beta=0\$ the solution approaches that of the Dupuit equation.

Solution for shallow slopes

For a shallow aquifer with ± 0 or the Dupuit conditions, the solution of the Boussinesq equation simplifies to

 $\label{eq:gw} = \left\{ GW \right\} = \left\{ \left\{ h^2_0 K_h \right\} \left\{ X \right\} \right\} \left\{ x \right\} \\ \left\{ a_{i=1}^{n} e_{n} \right\} \\ \left$

This equation represents a series of terms, the importance of terms for higher \$i\$ diminishes rapidly. If only the first term is retained, we get

 $q_{GW} = \left\{ 0, \frac{h^2_0*K_h}{X} \right\}$ right) * exp $\left[\frac{1.23*K_h*h_0}{X^2*S_y}*t \right]$

this is the equation for the drainage of a Dupuit aquifer.

Solution for steep slopes

A general solution of the equation is given by Brutsaert(1994). For a steep aquifer and large \$J\$, some simplifications can be introduced:

 $B = h^{S_y^{X}}$

where \$B\$ is the gravity drainable water. In addition the outflow can be described by Darcy's law as:

 $q_{GW}=h^{k}h^{tan beta}$

and hence (if no vertical recharge occurs:

 $q_{GW}=-\frac{dB}{dt}$

After substitution this leads to

 $sq_{GW}=K_h*h_0*tan beta * exp left (frac_K_h*tan beta_{S_y * X}*t right)$

The equation can be used to estimate the outflow from a sloping (steep and shallow) aquifer as a function of hydraulic conductivity \$K_h\$, thickness of the aquifer \$h_0\$, slope of the aquifer \$tan

\beta\$, storativity \$S_y\$ and length of the aquifer \$X\$ as a function of time \$t\$. This equation can be simplified to

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q_{GW}=a * h_0 * exp \left( \frac{-a}{S_y * X} \right)
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where $a=K_h*tan beta$.

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