Green & Ampt equation

The penetration depth of the infiltrating wetting front is Z at any moment in time t. If we assume that the wetting front is a sharp Dirac delta-function, Darcy's law can be stated as follows:

$$q = \frac{dI}{dt} = -K_s * \left[\frac{h_f-(h_s+Z)}{Z}\right]$$

where K_s is the hydraulic conductivity and $I(t)$ is the cumulative infiltration at time t that is equal to $Z*(\theta_s - \theta_0)$ (conservation of mass).

Using the above relation for $I(t)$ to eliminate Z and performing the integration yields,

$$I = K_s*t-(h_f-h_s)*(\theta_s - \theta_0)* log_e \left(1 - \frac{I}{(h_f-h_s)*(\theta_s-\theta_0)}\right)$$

with $I(t)$ infiltration amount in [cm], K_s hydr. conductivity in [cm/h], h_f wetting front pressure head (negative) in [cm], h_s water pressure at surface (ponding) in cm, θ_s moisture content at saturation, θ_0 antecedent moisture.

In order to solve this implicit equation, we need to bring $I(t)$ to one side of the equation:

$$\frac{1}{K_s}*\left[I -(h_f-h_s)*(\theta_s - \theta_0)* log_e \left(1 - \frac{I}{(h_f-h_s)*(\theta_s-\theta_0)}\right)\right] = t $$

We can then insert I can calculate t - we calculate the time that corresponds to a given infiltration amount. An R-code to calculate infiltration amounts with Green & Ampt looks like this:

|Green-Ampt.R

```
        I    <- seq(0,100,by=1.0)
        t0   <- 0.05
        ts   <- 0.25
        hs  <-  0.0 # cm
        hf  <- -12.0 # cm
        Ks  <-  8.0 # cm/hour
        t   <- 1/Ks*((I-(hf-hs)*(ts-t0))*log(1-(I/((hf-hs)*(ts-t0)))))
# hours
        plot(t,I,xlim=c(0,6),ylim=c(0,25),xlab="t [hour]", ylab="I in
[cm]")
```