Green & Ampt equation

At any time, t, the penetration of the infiltrating wetting front will be Z. Darcy's law can be stated as follows:

$$ \begin{equation*} q = \frac{dI}{dt} = -K_s * \left[\frac{h_f-(h_s+Z)}{Z}\right] \end{equation*} $$ where K_s is the hydraulic conductivity corresponding to the surface water content, and $I(t)$ is the cumulative infiltration at time t, and is equal to $Z*(\theta_s - \theta_0)$.

Using this relation for $I(t)$ to eliminate Z and performing the integration yields,

$$ \begin{equation*} I = K_s*t-(h_f-h_s)*(\theta_s - \theta_0)* log_e \left(1 - \frac{I}{(h_f-h_s)*(\theta_s-\theta_0)}\right) \end{equation*} $$ $$ \begin{table}

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with & $I(t)$ & infiltration amount & $[cm]$

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   &   $K_s$  & hydr. conductivity  & $[cm/h]$  \\
   &   $h_f$  & wetting front pressure head (negative) & $cm$  \\
   &   $h_s$  & water pressure at surface (ponding)    & $cm$  \\
   &   $\theta_s$ & moisture content at saturation     & $-$   \\
   &   $\theta_0$ & antecedent moisture                & $-$   \\
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\end{tabular} \end{table} $$ In order to solve this equation, we need to bring $I(t)$ to one side of the equation:

$$ \begin{equation*} \frac{1}{K_s}*\left[I -(h_f-h_s)*(\theta_s - \theta_0)* log_e \left(1 - \frac{I}{(h_f-h_s)*(\theta_s-\theta_0)}\right)\right] = t \end{equation*} $$

The R-program to calculate infiltration amounts with Green & Ampt looks like this:

|Greem-Ampt.R

```
        <<GreenAmpt, fig=TRUE, height=4.0, echo=FALSE>>=
        I    <- seq(0,100,by=1.0)
        t0   <- 0.05
        ts   <- 0.25
        hs  <-  0.0 # cm
        hf  <- -12.0 # cm
        Ks  <-  8.0 # cm/hour
        t   <- 1/Ks*((I-(hf-hs)*(ts-t0))*log(1-(I/((hf-hs)*(ts-t0)))))
# hours
        plot(t,I,xlim=c(0,6),ylim=c(0,25),xlab="t [hour]", ylab="I in
[cm]")
```